STATISTICAL ESTIMATION THEORY APPLIED TO SYNCHRONOUS GENERATOR MODELING

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SUMMARY

The purpose of this research is to model a synchronous generator, using statistical estimation theory to determine the parameters of the model from experimental data. An ideal generator model is reduced to a state-space formulation, and the method of quasilinearization is used to develop an optimal parameter estimation algorithm, implemented on a small digital computer.

The experiment to produce data for the parameter estimator is designed with the aid of a computer simulation of the experimental setup. The simulation includes a model of a typical generator with a switched load. The objectives of the design of the experiment are to produce sufficient data to permit the estimator to work well, while recognizing economic constraints on equipment and computer time.

The experiment thus designed, is implemented on a real synchronous generator with data recorded and digitized off-line. The data is then stored on a magnetic disk cartridge in a form suitable for use by the digital computer. The parameter estimates enable the generator to be modeled. The adequacy of the model is validated by predicting response of the generator to a larger change in load. The predicted response matches the actual response within the variance of the measurement noise.

The standard methods of machine parameter estimation are crude, requiring simplifying assumptions, and do not account for measurement

noise and inaccuracies. This dissertation, however, presents a method of estimating the generator model parameters from switched-load test data corrupted by considerable measurement noise.

CHAPTER I

INTRODUCTION

Motivation

A central problem facing power system analysts is the prediction of the dynamic behavior of the electric machinery connected to the system. The synchronous generators, which are universally used to convert mechanical energy to electrical energy in power plants, are of particular interest. Such machines can be simulated using digital computers if the parameters of mathematical models of the machines are known. Therefore, a problem of great practical importance to the electric power industry is the determination of the parameters of models of synchronous generators.

The parameters of an electrical machine can be calculated, at least in principle, if the internal geometry and material properties are known. However, such detailed information is often not available to the analyst. In this case, one must subject the machine to tests, recording data from measurements at the terminals. This data is invariably corrupted with noise during recording and processing. Therefore, the basic problem of determining machine parameters from tests is to develop an estimation algorithm to extract the parameters from noisy data and to design an experiment that produces sufficient data for this algorithm to work accurately.

Review of Past Approaches

A coupled circuit model of a three-phase synchronous generator derived by Park [1] is in general used by power system analysts. This model is obtained from a linear lumped-parameter model by transforming the armature quantities onto a two-axis coordinate frame that rotates with the rotor. Rotor circuits consisting of a field winding and two damper windings are invariant under the transformation. The result of Park's transformation is a set of stationary differential equations in the two-axis coordinate frame, related to the terminal voltages and currents by a set of time-varying measurement equations. The parameters of this model are the inductances and resistances of the two-axis circuits.

Since these parameters include mutual inductance between rotor and stator circuits and inductances of rotor circuits which are not accessible, a simplified parameter set is often used [2,3]. The simplified parameters can be determined from results of relatively simple tests [4]. An approximate analysis is carried out by assuming that a transient in the armature current initially affects the damper windings and then later affects the field winding. Also, the time constants of the dampers are assumed to be much shorter than the time constant of the field. These assumptions are generally accepted but tend to force the characteristics of a standard machine on all machines [2].

Several attempts to overcome the drawbacks of this simplified analysis have been published recently. Canay [5] added a parameter to account for a different coupling between the various rotor and armature circuits. The results predicted rotor quantities with greater accuracy. Yu and Moussa [6] then described several approaches for determining Canay's reactance from tests. Transfer functions for synchronous generators have been determined from sinusoidal perturbations about an operating point which are introduced by a fast-response static exciter [7] and from low-voltage measurements with the rotor at standstill [8]. The first method requires a source of field current (an exciter) which has a very fast response time while the second requires a variable frequency source of rather large current. In these frequency response tests, Bode plot construction techniques were used. This method requires judgement of the analyst in locating the breakpoint, since it is essentially a graphical method. No treatment of errors was given. Stanton [9] presented a statistical treatment of estimating a transfer function that was an empirical relation between rotor speed and electric power output. The result is not directly applicable to determining the parameters of a physical model. Lee and Tan [10] recently implemented a least-squares algorithm to estimate the parameters of the simplified analysis plus Canay's reactance. This approach assumed that the generator was subjected to a sudden three-phase short circuit test. The resulting estimator was tested on simulated data without the effects of measurement noise.

Definition of the Problem

The problem considered in this research is the estimation of the inductance and resistance parameters of Park's model of a synchronous generator. No intermediate parameter set requiring additional assumptions about the machine characteristics is imposed. Data is taken from the terminals of a synchronous generator under a resistive load. A transient is induced by a sudden switching of the load resistance to a smaller value. A statistical formulation of a parameter estimation algorithm is studied to enable an assessment of the effect of measurement noise upon the experiment. This approach overcomes many of the objections to previous approaches by applying estimation theory in a simple experiment to determine the parameters of a synchronous generator model directly.

A state-space approach is taken by casting the model in the form

$$\frac{dx}{dt} = f(x,y,u,t) \tag{1}$$

$$z = h(x,y,t,t) + w \quad , \tag{2}$$

where x is the state vector, u is the input vector, y is the parameter vector, z is the measurement vector and w is an additive noise term. The problem is to estimate y based on measurements z.

The first step is to derive a parameter estimator. This is an algorithm implemented on a digital computer for recursively computing estimates of the model parameters. A weighted least-squares approach is taken to minimize an error criterion that is the weighted sum of the square of the error. The error is defined as the difference between the observed output and the output computed from the model using the current parameter estimates. An optimal estimator is derived from this formulation by choosing the weighting matrix as the inverse of the noise covariance matrix.

The second step is to design an experiment which produces sufficient data to permit the estimator to work effectively. The success of the parameter estimator is directly dependent upon the experimental conditions. In particular, the load and excitation as well as the data sampling rate must be adequately chosen. To approach this problem, an experiment was designed with the aid of computer simulations. That is, a simulation of a generator with nominal parameters is used to produce data for the parameter estimator. This enabled the experimental conditions to be designed with maximum flexibility.

Finally, an experiment was implemented on an actual generator. Data recorded on an FM instrumentation tape recorder, was digitized and put into a form suitable for use by the estimation algorithm. The parameter estimator was then used to estimate the generator parameters. These estimates were used to predict the response of the generator to a larger change in load resistance. This final step validated the results and enabled the overall procedure to be evaluated.

CHAPTER II

DEVELOPMENT OF THE ESTIMATOR

Introduction

The object of this chapter is to develop the mathematical models needed to estimate the parameters of a synchronous generator. The approach taken is to model the generator, use this model to predict outputs based on the current parameter estimates, and then to adjust the parameter estimates systematically to minimize the weighted square of the difference between the measured outputs of the generator and the model outputs. This is the essence of a weighted least-squares estimation algorithm. Furthermore, incorporating the statistics of the noise which inevitably corrupts the measurements into the estimation algorithm leads to the maximum a posteriori probability estimator.

This chapter is divided into two main sections. The first presents the model of the generator in a form suitable for digital computer simulation. These equations are cast into a state-space notation for convenience. The second section develops weighted least-squares estimation algorithm by the method of quasi linearization. The statistics of the noise are used to derive an optimal estimator, which is the maximum a posteriori probability estimator. This formulation is a special case of weighted least-squares estimation with the weighting matrices determined from noise statistics and a priori information about parameter error statistics.

Generator Model

A typical three-phase, alternating-current generator consists of three armature windings placed symmetrically on the stator surrounding a rotor that is driven externally. The rotor consists of an iron core with a field winding excited by direct current. Additional rotor circuits called damper windings, consisting of short-circuited bars, are often imbedded in the rotor surface. Such a machine, considered here, is drawn schematically in Figure 1(a).

Due to the effect of the iron rotor, the armature inductances have components that vary with the rotor angle. By assuming symmetry of the rotor about the pole axis, or direct axis, and about the interpole axis (or quadrature axis) and by assuming sinusoidally distributed armature windings along the air gap, the fundamental component of the air gap flux linking the armature is proportional to $A + B\cos 2\theta$ [1]. As a result, the armature self and mutual inductances are

$$L_a = L_{a0} + L_{a1}\cos 2\theta \tag{3}$$

$$L_{ab} = -[L_{ab0} + L_{a1}\cos 2(\theta + \frac{\pi}{6})] \quad . \tag{4}$$

By projecting the armature circuit quantities onto the d-q coordinate frame, which is fixed in the rotor, Park [1] obtained a set of differential equations with constant coefficients. Fictitious windings along the d and q axes have constant inductances since the paths of flux have constant permeance. The self inductances of the model are

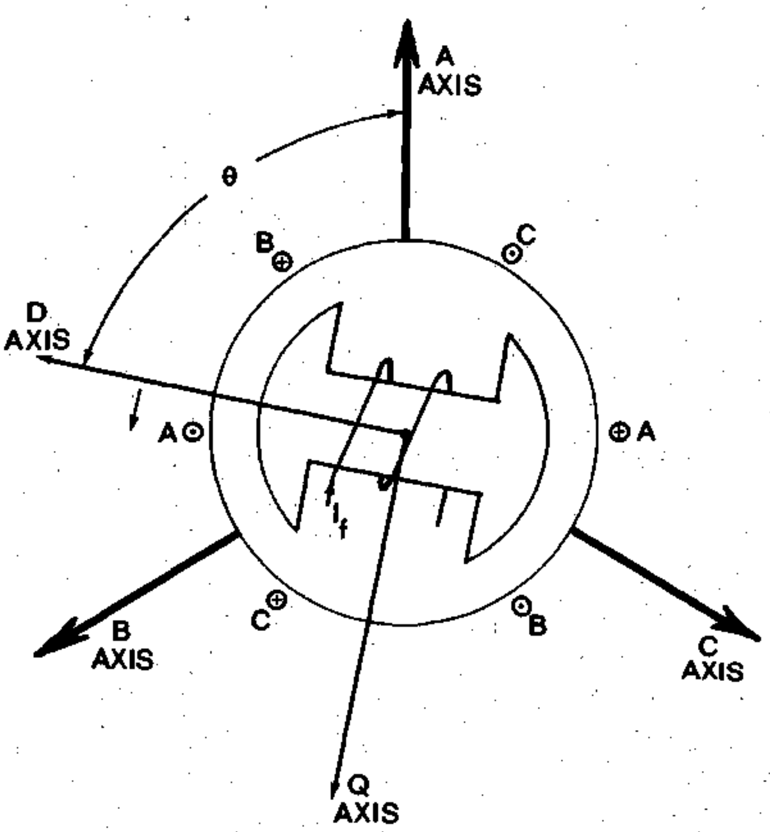


Figure 1a. Schematic Cross-Section of a Synchronous Generator.

$$L_D = L_{a0} + L_{ab0} + \frac{3}{2} L_{a1} \tag{5}$$

$$L_Q = L_{a0} + L_{ab0} - \frac{3}{2} L_{a1} \ . \tag{6}$$

Defining

$$L_1 \triangleq L_{a0} + L_{ab0} \tag{7}$$

and

$$L_2 \triangleq \frac{3}{2} L_{a1} \ , \tag{8}$$

then L_D and L_Q are computed from

$$L_D = L_1 + L_2 \tag{9}$$

$$L_Q = L_1 - L_2 \tag{10}$$

The net result of the preceding is the circuit model illustrated in Figure 1(b). It is described by a set of time-invariant, linear differential equations and a set of time-varying measurement equations. These are given by Equation (11) and Equation (12), respectively.

$$\frac{d\psi}{dt} = -(RL^{-1} + \Omega)\psi(t) + v(t) \tag{11}$$

$$z(t) = T(t)L^{-1}\psi(t) \tag{12}$$

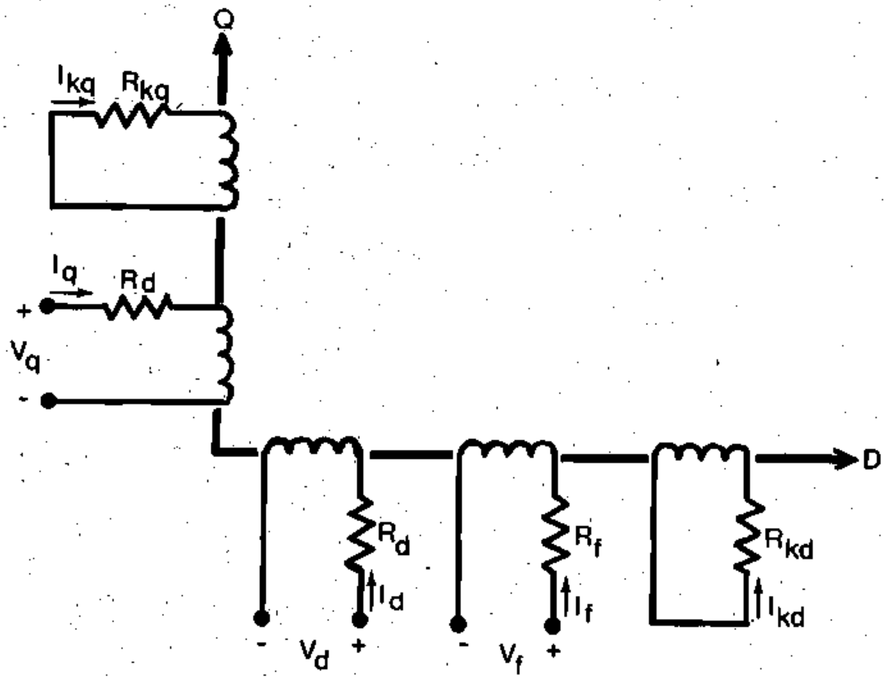


Figure 1b. Circuit Model of Synchronous Generator Due to Park.

where:

$\psi = (\psi_D\ \psi_F\ \psi_{KD}\ \psi_Q\ \psi_{KQ})^T$, the flux linkage vector

$v = (v_D\ v_F\ 0\ v_Q\ 0)^T$, the voltage vector

$z = (i_A\ i_B\ i_C\ i_F)^T$, the current vector

$$L = \begin{bmatrix} L_D & L_{DF} & L_{DKD} & 0 & 0 \\ L_{DF} & L_F & L_{FKD} & 0 & 0 \\ L_{DKD} & L_{FKD} & L_{KD} & 0 & 0 \\ 0 & 0 & 0 & L_Q & L_{QKQ} \\ 0 & 0 & 0 & L_{QKQ} & L_{KQ} \end{bmatrix}$$, the inductance matrix

$$R = \begin{bmatrix} R_D & & & & \\ & R_F & & 0 & \\ & & R_{KD} & & \\ & & & R_D & \\ & 0 & & & R_{KQ} \end{bmatrix}$$, the resistance matrix

$$\Omega = \begin{bmatrix} 0 & 0 & 0 & -\omega & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ \omega & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$T = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta & 0 & 0 & -\sin\theta & 0 \\ \cos(\theta - \frac{2\pi}{3}) & 0 & 0 & -\sin(\theta - \frac{2\pi}{3}) & 0 \\ \cos(\theta + \frac{2\pi}{3}) & 0 & 0 & -\sin(\theta + \frac{2\pi}{3}) & 0 \\ 0 & \sqrt{\frac{3}{2}} & 0 & 0 & 0 \end{bmatrix}$$

and $\theta = \omega t + \theta_0$. The measurement equations correspond to a transformation of coordinates from the D-Q reference frame to the three-phase armature reference frame, denoted by the subscripts A, B, and C. The rotor circuit quantities, denoted by subscript F for the field and by the subscripts KD and KQ for the damper windings, are invariant under this transformation.

These equations are not the same as those derived by Park [1] but have been modified as suggested by Lewis [11]. The result is that the transformation is unitary, that is the inverse of T is just the transpose T^T. The virtue of this particular transformation is that the mutual reactances are always reciprocal. This is not true of Park's original equations.

These equations are valid for balanced three-phase operation of the machine. If an unbalanced load is connected, a zero-sequence circuit must be added to the D and Q circuits to represent the armature circuits. The zero-sequence equations are defined by Equations (13), (14), and (15).

$$v_0 = \frac{1}{3}(V_a + V_b + V_c) \tag{13}$$

$$i_0 = \frac{1}{3}(i_a + i_b + i_c) \tag{14}$$

$$\psi_0 = \frac{1}{3}(\psi_a + \psi_b + \psi_c) \tag{15}$$

The zero-sequence variables are uncoupled from the rest of the equations, as shown by Equation (16).

$$\frac{d\psi_o}{dt} = -\frac{R_d}{L_o}\psi_o + v_o \tag{16}$$

Since the analysis and experiments will be carried out under balanced conditions, the zero-sequence equation is not considered further.

Generator parameters are usually expressed in per unit, that is in dimensionless ratios to base values. The base values are ordinarily chosen to be the rated values to facilitate comparison of machines of different sizes and ratings. Although Equations (11) and (12) are valid in any consistent set of units, such as the MKS system, a per unit system is used subsequently to simplify the computations and to be consistent with accepted practices.

Since $i = L^{-1}\psi$ and $p = \frac{d}{dt}$, the direct axis part of Equation (11) can be rewritten in operational form

$$\begin{bmatrix} v_D \\ v_F \\ 0 \end{bmatrix} = \begin{bmatrix} R_D + L_D p & L_{DF}p & L_{DKD}p \\ L_{DF}p & R_F + L_F p & L_{FKD}p \\ L_{DKD}p & L_{FKD}p & R_{KD}p \end{bmatrix} \begin{bmatrix} i_D \\ i_F \\ i_{KD} \end{bmatrix} + \begin{bmatrix} -\omega\psi_Q \\ 0 \\ 0 \end{bmatrix} . \tag{17}$$

Dividing each row by the corresponding base voltage and dividing and multiplying each column by the corresponding base current yields the equations in per unit.

$$\begin{bmatrix} v_d \\ v_f \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{i_B}{v_B}(R_D+L_Dp) & \frac{i_{FB}}{v_B}(L_{DF}p) & \frac{i_{KDB}}{v_B}(L_{DKD}p) \\ \frac{i_B}{v_{FB}}(L_{DF}p) & \frac{i_{FB}}{v_{FB}}(R_F+L_Fp) & \frac{i_{KDB}}{v_{FB}}(L_{FKD}p) \\ \frac{i_B}{v_{KDB}}(L_{DKD}p) & \frac{i_{FB}}{v_{KDB}}(L_{FKD}p) & \frac{i_{KDB}}{v_{KDB}}(R_{KD}+L_{KD}p) \end{bmatrix} \begin{bmatrix} i_d \\ i_f \\ i_{kd} \end{bmatrix} + \begin{bmatrix} -\omega\psi_q \\ 0 \\ 0 \end{bmatrix} \tag{18}$$

where

$$v_d = \frac{v_D}{v_B} \quad , \quad i_d = \frac{i_D}{i_B} \quad , \quad \psi_q = \frac{\psi_Q}{v_B}$$

$$v_f = \frac{v_F}{v_{FB}} \quad , \quad i_f = \frac{i_F}{i_{FB}} \quad , \text{ and } \quad i_{kd} = \frac{i_{KD}}{i_{KDB}} \quad .$$

Base current and voltage for the armature circuits are the rated values. Base field current is chosen to be the value of current which causes rated open-circuit armature voltage. Two constraints are placed on the three remaining arbitrary rotor base quantities,

$$v_{KDB} i_{KDB} = v_{FB} i_{FB} = v_B i_B \quad . \tag{19}$$

These relations imply that the base power is the same on all circuits and that the per unit inductance matrix is reciprocal. One more constraint will be placed on the base quantities of the damper winding circuit. Choosing

$$i_{KDB} = \frac{L_{DKD}}{L_{KD}} i_B \tag{20}$$

results in

$$\frac{L_{DKD}}{v_{KDB}} i_B = \frac{L_{KD}}{v_{KDB}} i_{KDB}$$

or

$$L_{dkd} = L_{kd} \quad , \tag{21}$$

in per unit. By similar reasoning, choice of $v_{KQB} i_{KQB} = v_B i_B$ results in a reciprocal quadrature axis inductance matrix and choice $i_{KQB} = \frac{L_{QKQ}}{L_{KQ}} i_B$ results in $L_{qkq} = L_{kq}$, in per unit. As a result, Equations (11) and (12) are valid with the following per-unit quantities:

$$L_d = \frac{L_D i_B}{v_B}, \quad L_f = \frac{L_F i_{FB}}{v_{FB}}, \quad L_{df} = \frac{L_{DF} i_{FB}}{v_B}, \quad L_{kd} = \frac{L_{KD} i_{KDB}}{v_{KDB}} = \frac{L_{DKD} i_B}{v_{KDB}}$$

$$L_{fkd} = \frac{L_{FKD} i_{KDB}}{v_{FB}}, \quad L_q = \frac{L_Q i_B}{v_B}, \quad L_{kq} = \frac{L_{KQ} i_{KQB}}{v_{KQB}} = L_{QKQ} \frac{i_B}{v_{KQB}},$$

$$R_d = \frac{R_D i_B}{v_B}, \quad R_f = \frac{R_F i_{FB}}{v_{FB}}, \quad R_{kd} = \frac{R_{KD} i_{KDB}}{v_{KDB}}, \text{ and } \quad R_{kq} = \frac{R_{KQ} i_{KQB}}{v_{KQB}};$$

$$L = \begin{bmatrix} L_d & L_{df} & L_{kd} & 0 & 0 \\ L_{df} & L_f & L_{fkd} & 0 & 0 \\ L_{kd} & L_{fkd} & L_{kd} & 0 & 0 \\ 0 & 0 & 0 & L_q & L_{kq} \\ 0 & 0 & 0 & L_{kq} & L_{kq} \end{bmatrix}, \quad R = \begin{bmatrix} R_d & & & & \\ & R_f & & 0 & \\ & & R_{kd} & & \\ & & & R_d & \\ & 0 & & & R_{kq} \end{bmatrix};$$

$$\psi_q = \frac{\psi_Q}{v_B}, \quad \psi_d = \frac{\psi_D}{v_B}, \quad \psi_f = \frac{\psi_F}{v_{FB}}, \quad \psi_{kd} = \frac{\psi_{KD}}{v_{KDB}}, \quad \psi_{kq} = \frac{\psi_{KQ}}{v_{KQB}}$$

Equation (17) has two more parameters than the per unit equations since the damper winding currents are scaled by a factor containing a ratio of damper winding inductances. This choice of base currents results in a reduction in the number of inductance parameters and the addition of an equal number of parameters in the base rotor quantities. The object of this model is to represent the generator by a circuit equivalent at the terminals. Since the damper windings are inaccessible short-circuited turns imbedded in the rotor, the current in them does not need to be determined absolutely to model the generator at the terminals. The net result of this particular per unit representation is that two parameters are not determined but the resulting circuit is still equivalent at the armature and field terminals.

Since the experiment was conducted with the generator under a balanced resistive load, as described in Chapter III, this special case is now considered. The field is excited by a regulated voltage source providing a constant v_f. The components of the armature voltage are proportional to the corresponding currents.

$$v_d = -R_L i_d$$

$$v_q = -R_L i_q \tag{23}$$

Therefore, Equation (11) still holds if R_d is replaced by $R_d + R_L$, and if $v = (0 \quad v_f \quad 0 \quad 0 \quad 0)^T$.

For convenience in the following derivation, the model Equations (11) and (12) are written in more general notation.

$$\frac{dx}{dt} = f(x,y,v) = F(y)x + v; \qquad x(0) = x_o \tag{24}$$

$$z(t) = h(x,y,\theta_o t) \tag{25}$$

where

$x \;=\; x(x_o,y,v,t)$ = state vector (flux linkages)

$y \;=\;$ parameter vector

v = input vector (field voltage)

t = time

θ_o = $\theta_o(x_o)$ = initial rotor angle

x_o = $x_o(y)$ = initial state vector

z = measurement vector (terminal currents)

The matrix $F = -(RL^{-1} + \Omega)$ and the parameter vector is

$$y = (L_1\ L_{df}\ L_{kd}\ L_f\ L_{fkd}\ L_2\ L_{kq}\ R_d\ R_f\ R_{kd}\ R_{kq})^T \tag{26}$$

where

$$L_d = L_1 + L_2$$
$$L_q = L_1 - L_2 \tag{27}$$

These equations assume the generator is in steady state at time $t=0$. Therefore, the initial state vector x_o and the initial rotor angle θ_o are computed from the parameters by the relations

$$x(0) = x_o(y) = -F(y)^{-1}v \tag{28}$$

and

$$\theta_o = \theta_o(x_o) = \arctan\left(\left.\frac{id}{iq}\right|_{t=0}\right) = \arctan\left(\frac{\omega L_q}{Rd+RL}\right) . \tag{29}$$

For $t>0$, a transient is induced by a step change in load resistance and the state vector is found by numerical solution of Equation (24).

Equations (24) and (25) describe a continuous-time, deterministic state-space model of the generator. Since a digital computer estimation algorithm is developed in the sequel, consider the measurement Equation (25) to be a function of t_k for $k = 1,2, \ldots, k_f$, a set of discrete time points. Finally, since inaccuracies and noise inevitably corrupt the measurement process, a term representing an additive measurement noise sequence is included. The result is expressed in Equation (30).

$$z(t_k) = h(x,y,\theta_o,t_k) + w_k \quad , \quad k = 1,2, \ldots, k_f \tag{30}$$

where t_k is the k^{th} discrete time point, and w_k is the k^{th} discrete noise sample.

<u>Weighted Least-Squares Estimation</u>

Consider a parameter estimator, an algorithm not yet specified, which recursively generates an estimate of the parameter vector. If the current parameter estimate is denoted by $\hat{y}$, then the output from a model of the generator is $h(\hat{x},\hat{y},\theta_o,t_k)$, where $\hat{x}$ is the state vector estimate computed from numerical solution of the model Equation (24) using $\hat{y}$. If the measured output of the actual generator is $z(t_k)$, then the weighted least-squares error criterion is given by

$$J = \frac{1}{2} \sum_{k=1}^{k_f} \{[z(t_k)-h(\hat{x},\hat{y},\theta_o,t_k)]^T Q [z(t_k)-h(\hat{x},\hat{y},\theta_o,t_k)]\} \quad . \tag{31}$$

Here Q is a non-negative definite weighting matrix. The interpretation of this error criterion is that minimizing J also minimizes the weighted squares of the error between the observed output and the modeled output.

In the estimation algorithm, the method of quasilinearization [12,13] will be applied. Let the current parameter estimate $\hat{y}$ be updated to produce the new estimate

$$\hat{y}_{new} = \hat{y} + \Delta y \quad . \tag{32}$$

Expanding equation (30) in a Taylor series about $\hat{y}$ and retaining only the first two terms gives the linear approximation

$$z(t_k) \cong \hat{h}_k + \frac{d\hat{h}_k}{dy} \Delta y + w_k \tag{33}$$

where

$$\hat{h}_k = h(\hat{x},\hat{y},\theta_o,t_k)$$

and

$$\frac{d\hat{h}_k}{dy} = \left.\frac{dh(x,y,\theta_o,t_k)}{dy}\right|_{\hat{x},\hat{y}} \quad .$$

Repeated application of the chain rule for partial derivatives give

$$\frac{dh}{dy} = \frac{\partial h}{\partial y} + \frac{\partial h}{\partial x}\left(\frac{\partial x}{\partial y} + \frac{\partial x}{\partial x_o}\,\frac{dx_o}{dy}\right) + \frac{\partial h}{\partial \theta_o}\,\frac{d\theta_o}{dx_o}\,\frac{dx_o}{dy} \tag{34}$$

Differentiating Equation (24) with respect to y and x_o results in the sensitivity matrix differential equations (35) and (36).

$$\frac{d}{dt}\left(\frac{\partial x}{\partial y}\right) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial f}{\partial y} \quad , \quad \left.\frac{\partial x}{\partial y}\right|_{t=0} = \frac{\partial x_o}{\partial y} \tag{35}$$

$$\frac{d}{dt}\left(\frac{\partial x}{\partial x_o}\right) = \frac{\partial f}{\partial x}\frac{\partial x}{\partial x_o} \quad , \quad \left.\frac{\partial x}{\partial x_o}\right|_{t=0} = I \ . \tag{36}$$

The solutions to (35) and (36) are the sensitivity matrices

$$\frac{\partial x}{\partial y} \text{ and } \frac{\partial x}{\partial x_o} \ . \tag{37}$$

A necessary condition for minimizing J is that its gradient with respect to y vanish. Evaluating the gradient at the new parameter estimate $\hat{y} + \Delta y$ results in

$$\left.\frac{dJ}{dy}\right|_{\hat{y}+\Delta y} = 0 = -\sum_{k=1}^{k_f}\left(\frac{d\hat{h}_k}{dy}\right)^T Q\{z(t_k) - [\hat{h}_k + \frac{d\hat{h}_k}{dy}\Delta y]\}) \tag{38}$$

Solving for Δy gives

$$\Delta y = \left[\sum_{k=1}^{k_f} \frac{d\hat{h}_k}{dy}^T Q \frac{d\hat{h}_k}{dy}\right]^{-1} \sum_{k=1}^{k_f}\left\{\frac{d\hat{h}_k}{dy} Q[z(t_k) - \hat{h}_k]\right\} \ . \tag{39}$$

Setting the new estimate equal to $y + \Delta y$ completes the recursive algorithm for computing the parameter estimates.

In summary, the algorithm consists of solving the differential Equations (24), (35) and (36) numerically, computing sensitivity matrices and solving the system of linear algebraic Equations (39).

This recursive algorithm is ideally suited for implementation on a digital computer.

Statistical Estimation Theory

The least-squares approach of the previous section ignores the statistical nature of the estimation problem. If information about the statistics of the noise is available, the estimation process can be improved. This section presents a derivation and discussion of relevant aspects of stochastic parameter estimation theory.

Bayesian Approach

Consider an error defined to be

$$y - \hat{y}(z) \tag{40}$$

where $\hat{y}(z)$ is some estimate based on measuring z. Let $C(y-\hat{y}(z))$ be the cost function describing the penalty for making that error. The Bayesian risk [14,15] is defined as the conditional mean of the error

$$R = E\{C(y - \hat{y})|z\} = \int_{-\infty}^{\infty} C(y - \hat{y})p(y|z)dy \quad . \tag{41}$$

For the case of the uniform cost function,

$$C(y-\hat{y}) = \begin{cases} \frac{1}{\epsilon} & \text{if } \|y-\hat{y}\| \geq \epsilon \\ 0 & \text{otherwise} \end{cases} \quad . \tag{42}$$

Consider the limit of the cost function as $\epsilon \to 0$,

$$C(y - \hat{y}) = -\prod_{j=1}^{N} \delta(y_j - \hat{y}_j) \quad . \tag{43}$$

As a result

$$R = -p(\hat{y}(z)|z) \quad . \tag{44}$$

Minimizing the Bayesian risk is equivalent to maximizing the posterior probability density.

Maximum A Posteriori Probability Estimator

Denote the set of all measurement vectors in the sequence by $Z = \{z(t_1), z(t_2), \ldots, z(t_f)\}$. If the measurement noise is represented as a vector stochastic process and the parameter vector as a random vector, the posterior probability density is given by Bayes' rule as

$$p(y|Z) = \frac{p(Z|y)p(y)}{p(Z)} \quad . \tag{45}$$

Since $p(Z)$ is not dependent on y, maximizing (45) by choice of y is equivalent to maximizing $p(Z|y)p(y)$.

If the measurement noise is normally distributed with zero mean and covariance matrix V_w, the conditional probability of the k^{th} measurement vector is given by

$$p[z(t_k)|y] = [(2\pi)^m \det(V_w)]^{-\frac{1}{2}} \cdot \exp\{-\tfrac{1}{2}[z(t_k) - h(x,y,t_k)]^T V_w^{-1} [z(t_k) - h(x,y,t_k)]\} \tag{46}$$

If the noise samples are uncorrelated with each other, that is from a white noise sequence, the probability of observing the entire sequence Z is simply the product of k_f terms like the right side of Equation (46).

$$p[Z|y] = [(2\pi)^m \det(V_w)]^{-k_f/2}$$

$$\cdot \exp\{-\tfrac{1}{2}\sum_{k=1}^{k_f}[z(t_k)-h(x,y,t_k)]^T V_w^{-1}[z(t_k)-h(x,y,t_k)]\} \tag{47}$$

If the random parameter vector y is now assumed to be normally distributed with mean M_y and variance V_y, then

$$p(y) = [(2\pi)^m|V_y|]^{-\frac{1}{2}}\exp[-\tfrac{1}{2}(y-M_y)^T V_y^{-1}(y-M_y)] \quad . \tag{48}$$

Since the exponential function is monotonic in its argument and since the functions multiplying the exponentials in Equation (47) and (48) do not contain y, maximizing $p(Z|y)p(y)$ is equivalent to minimizing the error criterion

$$J = \tfrac{1}{2}\sum_{k=1}^{k_f}[z(t_k)-h(x,y,t_k)]^T V_w^{-1}[z(t_k)-h(x,y,t_k)]$$

$$+ \tfrac{1}{2}(y-M_y)^T V_y^{-1}(y-M_y) \quad , \tag{49}$$

resulting in

$$\Delta y = [\sum_{k=1}^{k_f} (\frac{d\hat{h}_k}{dy}^T V_w^{-1} \frac{d\hat{h}_k}{dy}) + V_y^{-1}]^{-1} \{ \sum_{k=1}^{k_f} [\frac{d\hat{h}_k}{dy}^T V_w^{-1} (z(t_k) - \hat{h}_k)] - V_y^{-1} (\hat{y} - M_y) \} \ . \tag{50}$$

Thus the maximum a posteriori probability estimator can be considered a special case of a least-squares estimator, including prior information about the parameter vector, if the weighting matrices are chosen equal to the inverse of the error covariance matrices.

Lower Bound on the Error Covariance

A lower bound on the error covariance of the estimator derived is easily obtained from a generalization of Fisher's information matrix [15,16].

$$E[(y-\hat{y})(y-\hat{y})^T] \leq \{ \sum_{k=1}^{k_f} [\frac{d\hat{h}_k}{dy}^T V_w^{-1} \frac{d\hat{h}_k}{dy}] + V_y^{-1} \}^{-1} \tag{51}$$

The derivation of this bound is presented in Appendix A. This lower bound is computationally inexpensive since the right side of Equation (51) is already computed in the estimation algorithm, Equation (50).

Summary

The derivation of an algorithm suitable for estimating synchronous generator parameters is approached from the point of view of weighted least-squares estimation theory. First, the equation describing the generator are cast into a state-variable form. Next, the method of quasilinearization is used to derive a least-squares estimation algorithm. By considering a stochastic formulation, the maximum a posteriori probability estimator is shown to be a special case of the weighted least-squares algorithm with correct choice of the weighting matrices.

While the direct implementation of the weighted least squares algorithm results from weights chosen arbitrarily or by physical intuition, the optimal estimator improves the quality of the estimates by using the prior error statistics to choose the proper weights. Unfortunately, the exact statistics of the noise are not known. The approach taken here is to study the effect of incorrect prior statistics using a computer simulation. Then, when processing data from the actual generator, estimates of the relative magnitudes of error variances can be made to enable intelligent choice of weights. If these estimates of the prior statistics are correct, the result should approach the performance of the optimal estimator. If the statistics are in error, at least the resulting suboptimal estimate satisfies the least-squares error criterion. In any case, the algorithm minimizes the weighted square of the output error.

CHAPTER III

DESIGN OF THE EXPERIMENT

Introduction

The success of an iterative parameter estimation algorithm, such as the one previously described, depends on the quality of the input data. In other words, the experiment that produces this data must be designed with the criterion as the performance of the estimator. In this research the experiment was designed with the aid of computer simulations. This enabled various experimental conditions to be tested with maximum flexibility.

First, certain constraints on available equipment and on computing and processing time must be recognized. These are primarily economic limitations. The experiment was designed within these constraints by selecting such factors as machine load and excitation, data sampling rate and overall data record length. To choose these conditions intelligently, a computer simulation of a typical generator was used to model the experimental setup. The resulting test data was used to assess the performance of the estimator. Therefore, the effect of the undetermined experimental condition was easily established and the experiment thereby designed.

Equipment Constraints

The test generator was a three-phase, four-pole, alternating current, synchronous machine rated at 3 KVA and 230 volts at 40 hertz.

The drive motor was a 15 horsepower direct current machine rated at 1200 revolutions per minute. The dc supply, from a large motor-generator set, had only open-loop control. As a result, the speed of the drive motor was manually adjustable with no provision for automatic regulation.

The load bank consisted of three power resistors connected in wye with an additional resistor suddenly switched in parallel with each leg to induce a transient. This arrangement is illustrated in Figure 2. The closing of the three-pole switch, labeled S, causes the load resistance to suddenly decrease from R_1+R_3 to $\frac{R_1R_2}{R_1+R_2} + R_3$. Resistors R_3 are current shunts used to obtain voltage signals proportional to the load current in each leg. These signals, along with a similar one proportional to field current, were recorded and processed as described in the next chapter.

Excitation of the field was supplied by a regulated electronic dc power supply rated at 0.5 amp and 500 volts. This was adequate to supply up to the rated field current of .525 amps at the rated field voltage of 125 volts.

The main constraint posed by the available equipment was to limit armature current to values within the generator ratings to prevent damage to the windings. The second constraint was a limit on the maximum change in armature current during the test. To insure that these constraints were met, the field voltage was reduced below its rated value, reducing the magnitude of the load currents.

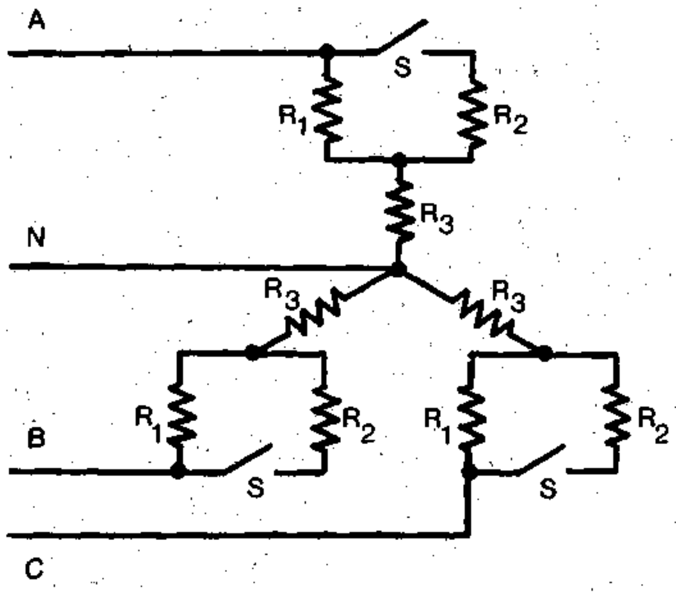


Figure 2. Schematic Diagram of Load Bank.

Simulation Using Nominal Parameters

Within the constraints already posed, several experimental conditions remain to be selected. First, the magnitude of the step load change must be selected large enough to enable all the parameters to be estimated. Some parameters, notably the damper circuit reactances and resistances, affect the terminal currents only during transients. Thus a significant transient must be introduced by the sudden load changes. Second, the data sampling rate must be chosen fast enough to accurately represent the terminal current waveforms. The well-known sampling theorem states that the minimum rate is twice the highest frequency component of the waveform. Experience shows that the practical minimum is somewhat faster than the theoretical limit. A balancing constraint is the limitation of the total amount of data storage space available. Finally, the length of the data record and the number of load switchings must be determined.

To determine these conditions so that the experiment would be successful, the experimental setup was modeled with a computer simulation. The machine was modeled with Equations (24) through (30) solved numerically using a modified Runge-Kutta method [17]. The modification, due to Merson [18], allowed an estimate of the roundoff error to be computed to insure the step size was small enough. A multiplicative type pseudo-random noise generator [19] simulated additive measurement noise. The simulation, implemented on a small digital computer with a magnetic disk operating system, provided test outputs to allow assessment of the effect of the experimental conditions on the parameter estimator. This information allows the experiment to be designed.

To implement the numerical solution of the machine model, a set of typical parameters was determined from nameplate data, a few simple tests, and a list of typical machine constants [3]. The parameters X_d, X_{df}, R_d, and R_f were measured by simple ac and dc steady-state tests. The remaining parameters were roughly estimated from the typical parameter list. The results of this nominal parameter computation are given in Table 1. The results of the steady-state tests and the details of the calculation are presented in Appendix B. It should be emphasized that the nominal-parameter model of the machine is not an accurate representation of this generator but is similar to a typical generator.

The first run of the simulation modeled the generator during a sudden three-phase short circuit on the armature terminals. The main purpose of this run was to test the operation of the estimator. No measurement noise was added and the measurements were weighted equally. The step size was 2 msec. and the field voltage was the rated value. The parameter estimates converged rapidly as shown in Figure 3(a) and (b), which are plots of the measured relative error versus iteration. The relative error is defined as

$$\epsilon = \frac{\hat{y}-y}{y} \tag{52}$$

where $\hat{y}$ is the parameter estimate, and y is the actual parameter value. Figure 3(c) shows the current in phase a and in the field versus time. The outputs based on the parameter estimates match the data within graphical error.

Table 1. Nominal Parameters in Per Unit

L_d	=	3.25×10^{-3}
L_{kd}	=	2.20×10^{-3}
L_{fkd}	=	4.01×10^{-3}
L_{kq}	=	1.17×10^{-3}
R_f	=	2.31×10^{-2}
L_{df}	=	4.87×10^{-3}
L_f	=	10.8×10^{-3}
L_q	=	2.12×10^{-3}
R_d	=	1.42×10^{-2}
R_{kd}	=	7.59×10^{-2}

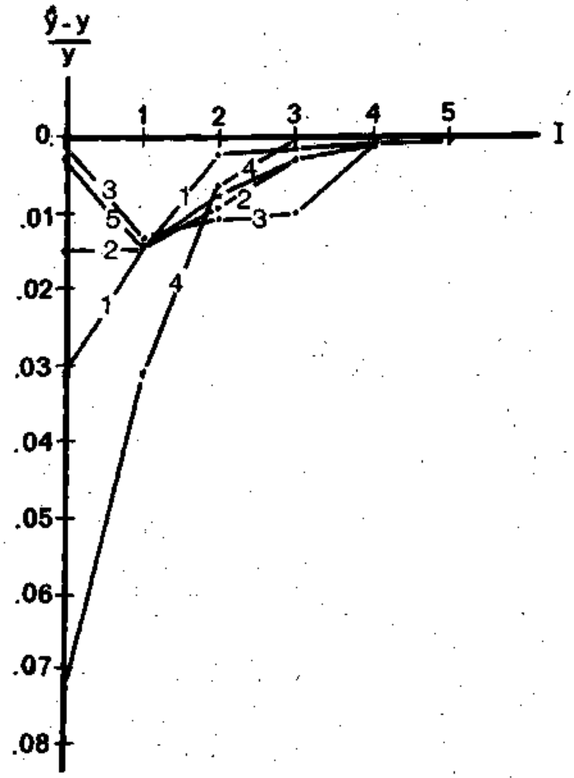


Figure 3a. Parameter Error for Sudden Short-Circuit, Test One, Parameters 1-5.

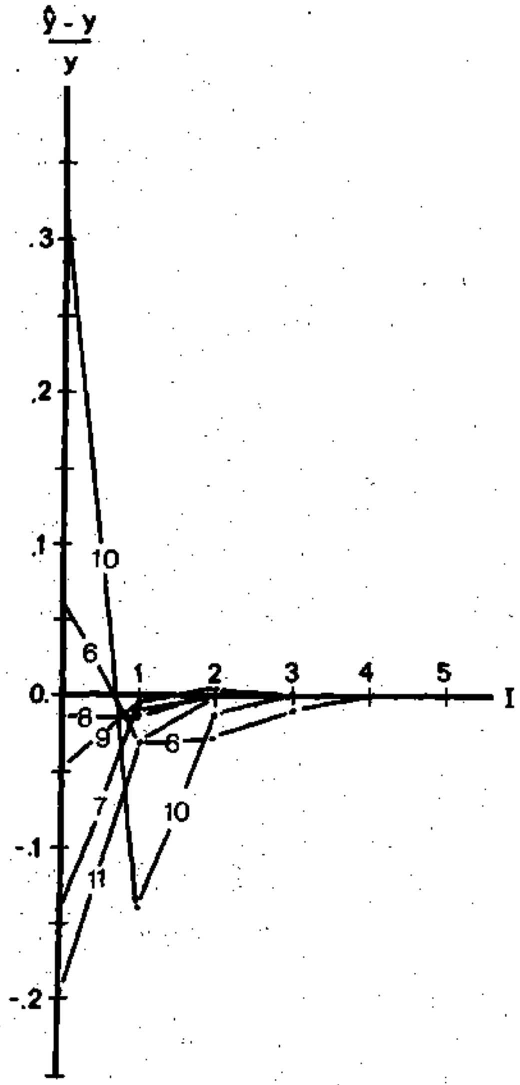
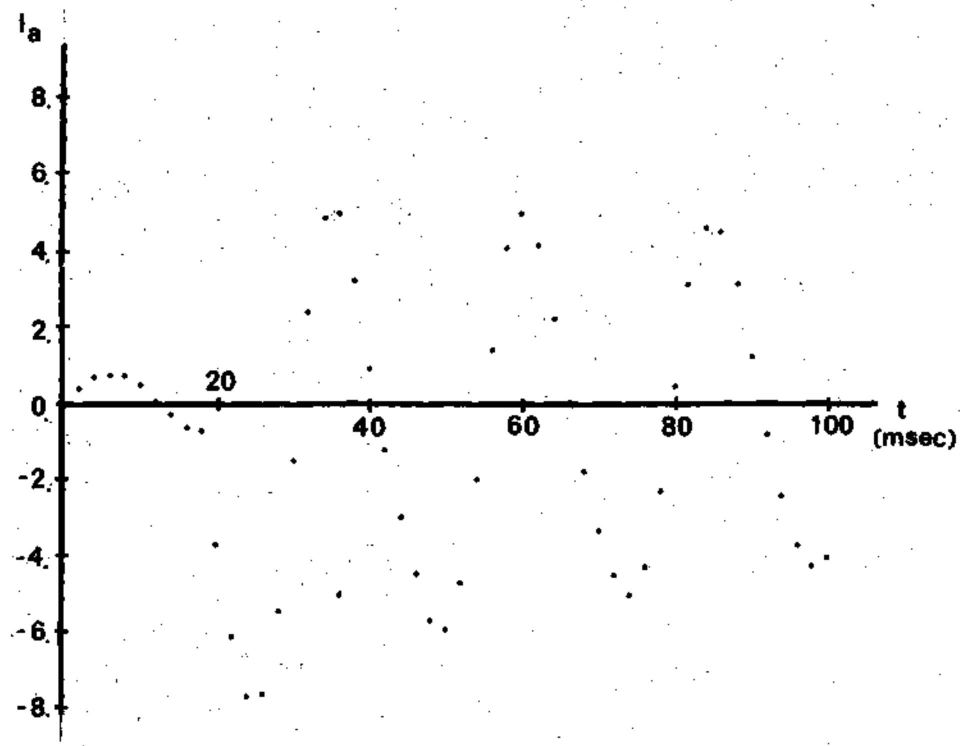


Figure 3b. Parameter Error for Sudden Short-Circuit, Test One, Parameters 6-11.



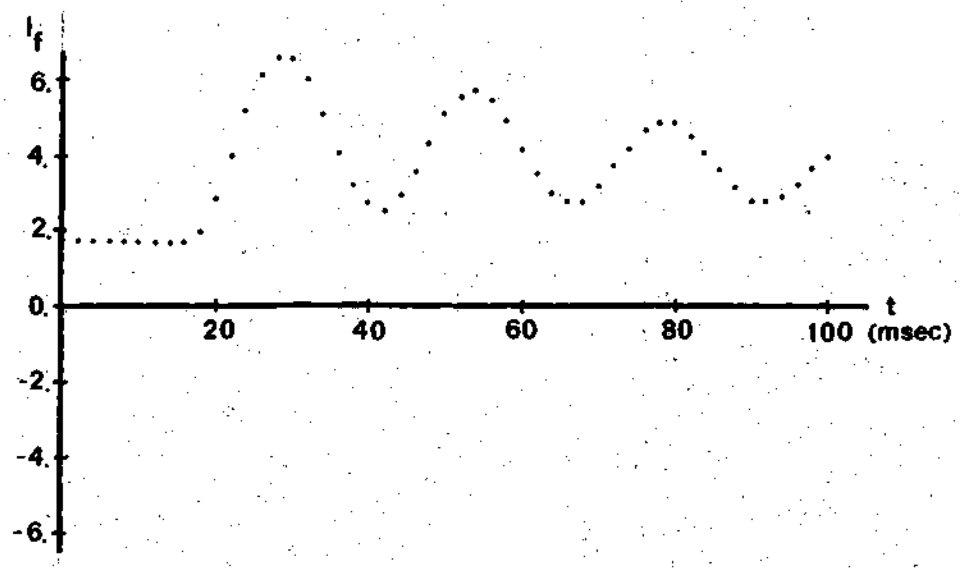


Figure 3c. Armature and Field Current for Sudden Short-Circuit.

The second test is similar to the first; however, a transient was induced by switching load resistance rather than a sudden short circuit. The switched load test is a more realistic simulation of the actual experimental setup. With the machine initially in steady state, the load resistance was suddenly switched from 1.0 to 0.25 per unit. The load was switched back to one per unit, 400 milliseconds later.

Convergence of the estimator was somewhat slower than in the previous test, as shown in Figure 4(a) and (b). Figure 4(c) shows the terminal currents computed with the parameter estimates after ten iterations to be quite close to the correct values. The errors remaining in the estimates, then, have small effect on the terminal currents.

Tests run with switched loads of differing magnitudes show that smaller resistances, approaching the short-circuit test in the limit, produce results closer to those obtained in test one. On the other hand, resistances approaching 1.0 lead to smaller transients and poorer results.

Tests in which one transient is induced by load switching show poorer results than the previous test with two transients. Two load switchings, from normal load to small load to normal load, provide redundant information. This aids the operation of the estimator, particularly in the presence of noise. More than two load switchings are not considered, because more than two quick operations of the three-pole switch are not practical. The experiment then was constrained to two load resistance switchings.

Further tests with different switching times showed essentially

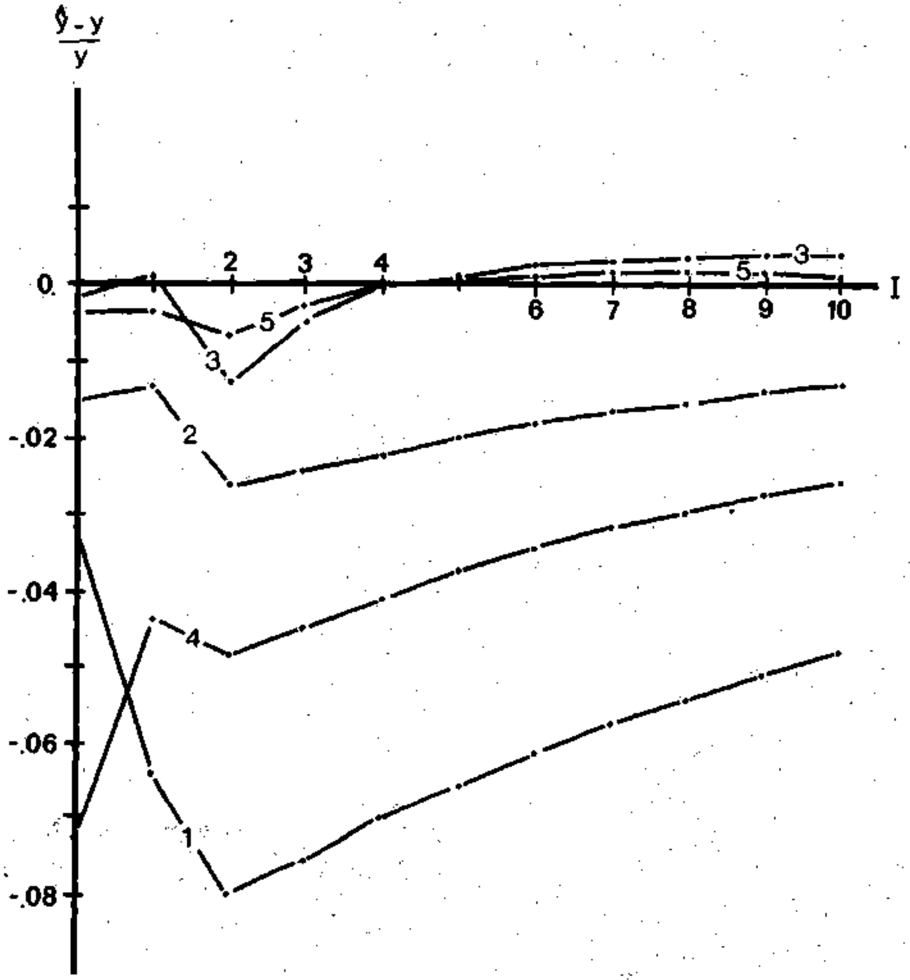


Figure 4a. Parameter Error for Switched Resistive Load, Test Two, Parameters 1-5.

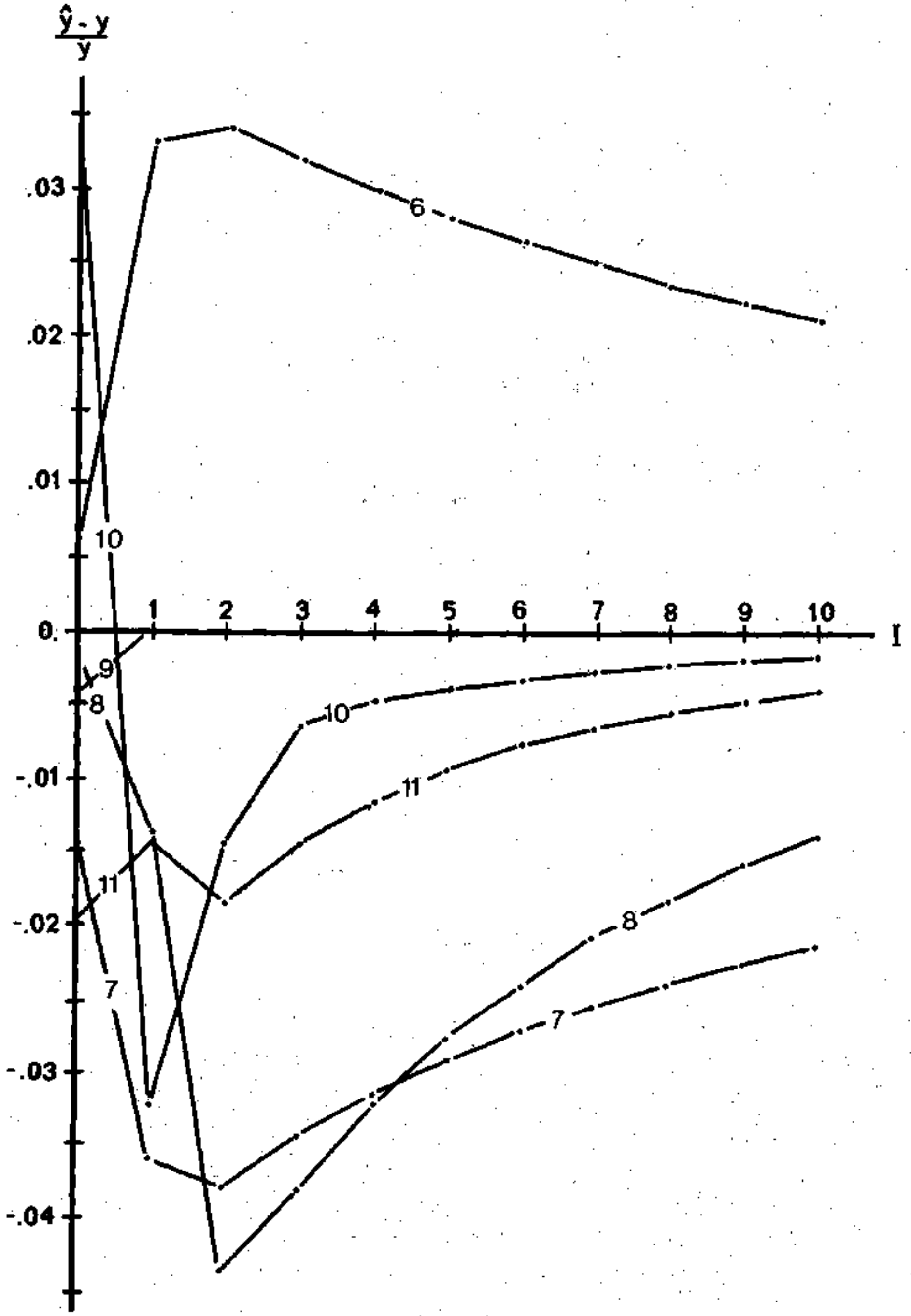


Figure 4b. Parameter Error for Switched Resistive Load, Test Two, Parameters 6-11.

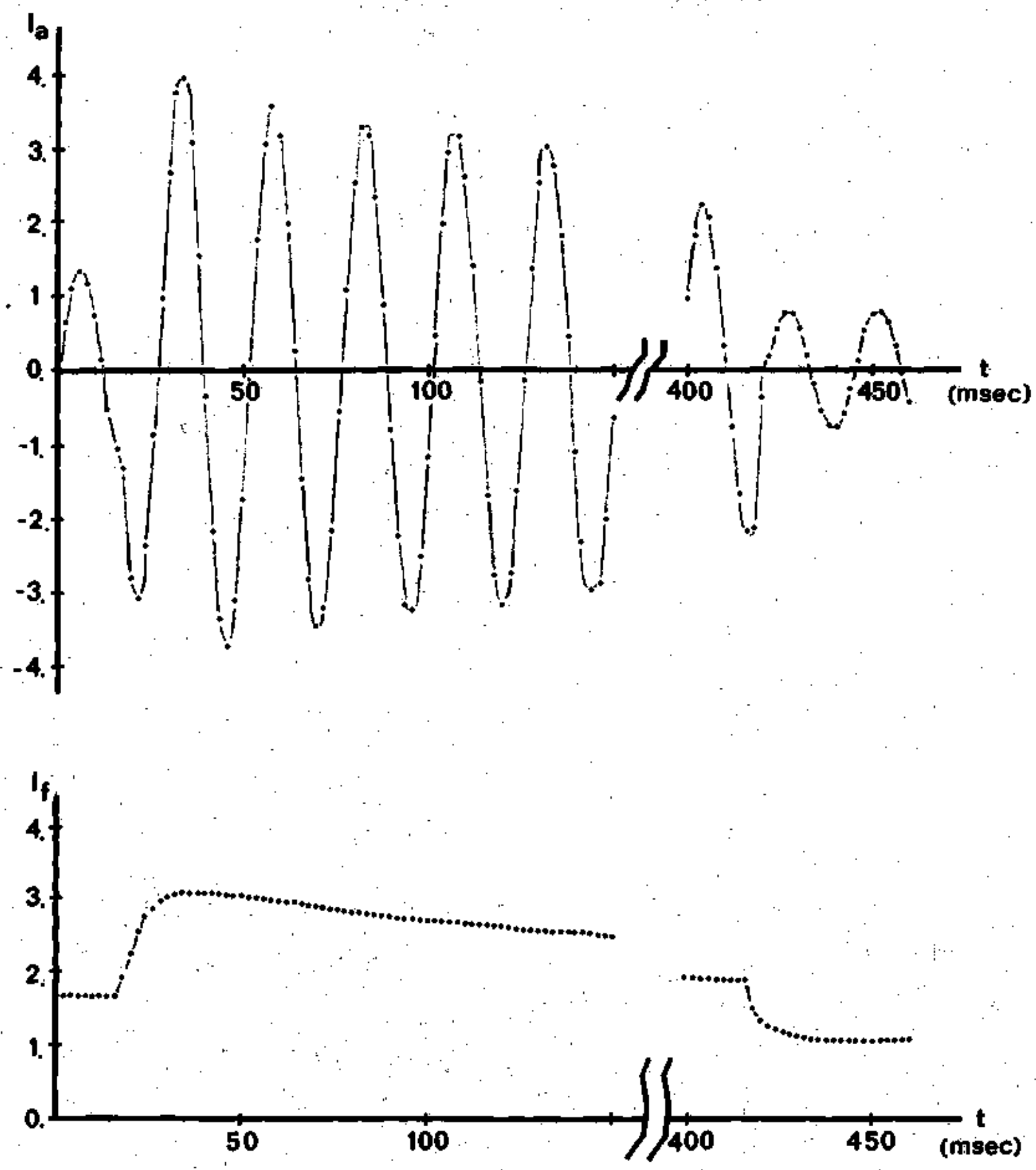


Figure 4c. Armature and Field Current for Switched Resistive Load, Test Two.

the same behavior. The minimum time for operating the switch prohibited very short durations, and the limitations on overall data record length prohibited very long durations. For manual operation of the switch, a duration of several hundred milliseconds is reasonable.

The next test, illustrated in Figure 5, shows the effects of ten percent errors in the initial parameter guesses and measurement noise with zero mean and standard deviation $1x10^{-2}$. All measurements and parameters were weighted equally.

The effect of increasing measurement noise is to degrade the performance of the estimator. At noise of standard deviation greater than 1.0, the estimator does not converge. With noise of standard deviation of 0.1, the estimator converged very slowly, but large errors in some parameters show that they tend to track the noise.

The final test with simulated data, shown in Figure 6, was the same as the preceeding test; however, the weighting matrices were chosen as the inverse covariance matrices. This is an implementation of the maximum a posteriori probability estimator. The results show only minor improvements on the preceeding weighted least-squares approach, shown in Figure 5.

The results of the simulated experiment show that the estimator will work satisfactorily with a resistive load switched from 1.0 to 0.25 to 1.0 per unit, with practical switching periods, in the presence of moderate initial parameter errors and measurement noise. This experiment was consistent with the limitations imposed by available equipment. The implementation of this experiment is discussed in the next chapter.

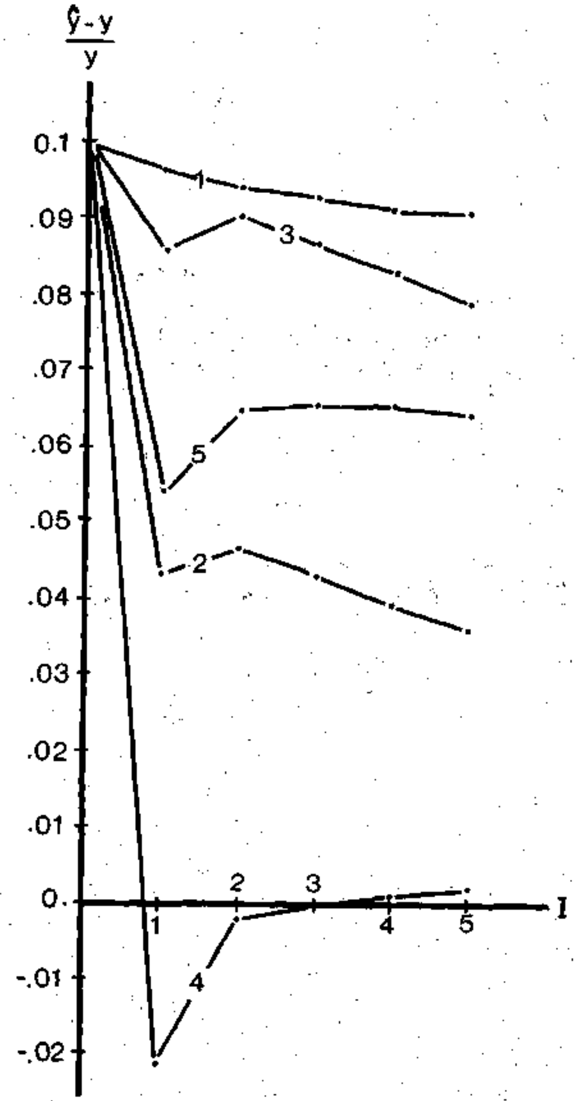


Figure 5a. Parameter Error Versus Iteration for Test Three, Parameters 1-5.

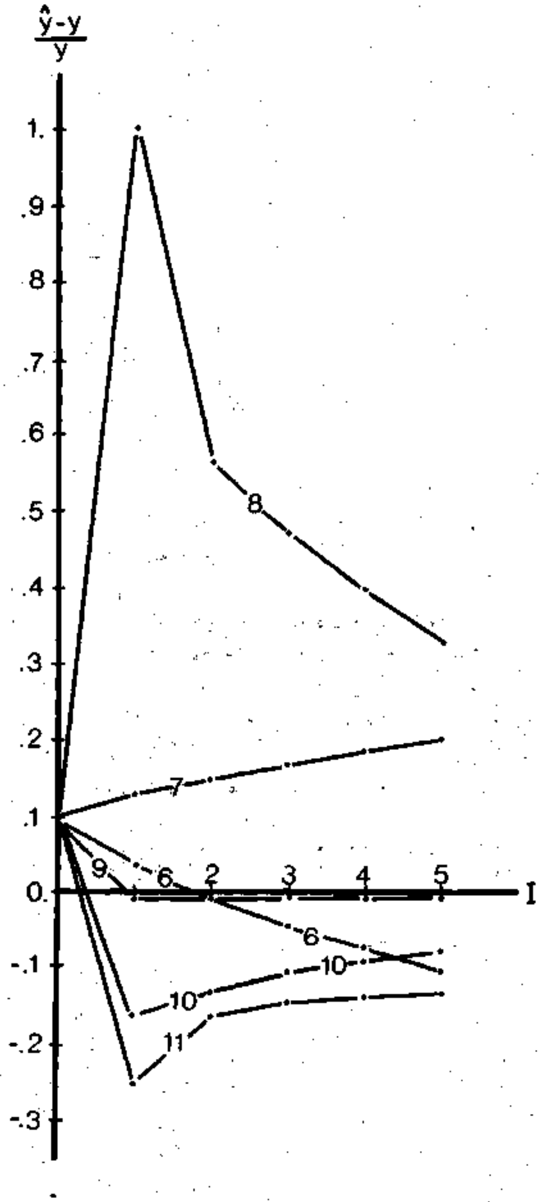


Figure 5b. Parameter Error Versus Iteration for Test Three, Parameters 6-11.

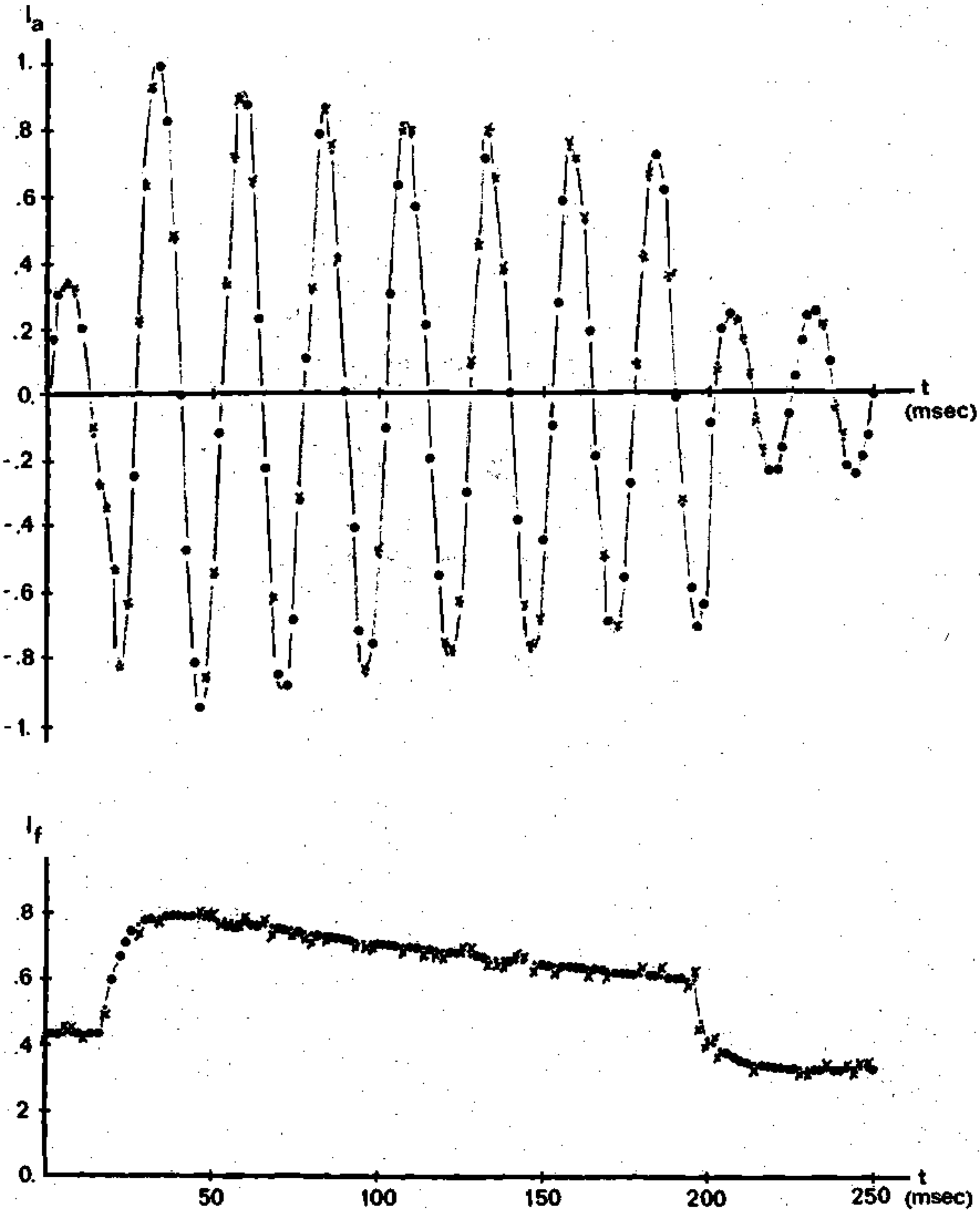


Figure 5c. Armature and Field Current for Test Three

- • = result of estimates
- × = input data
- ● = points where estimated currents equal data within graphical error

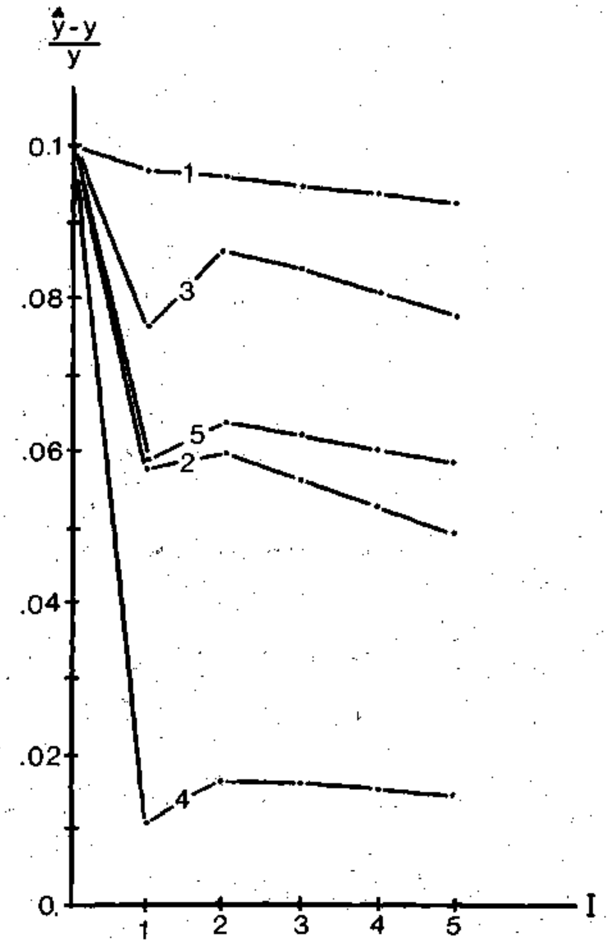


Figure 6a. Parameter Error Versus Iteration for Test Four, Parameters 1-5.

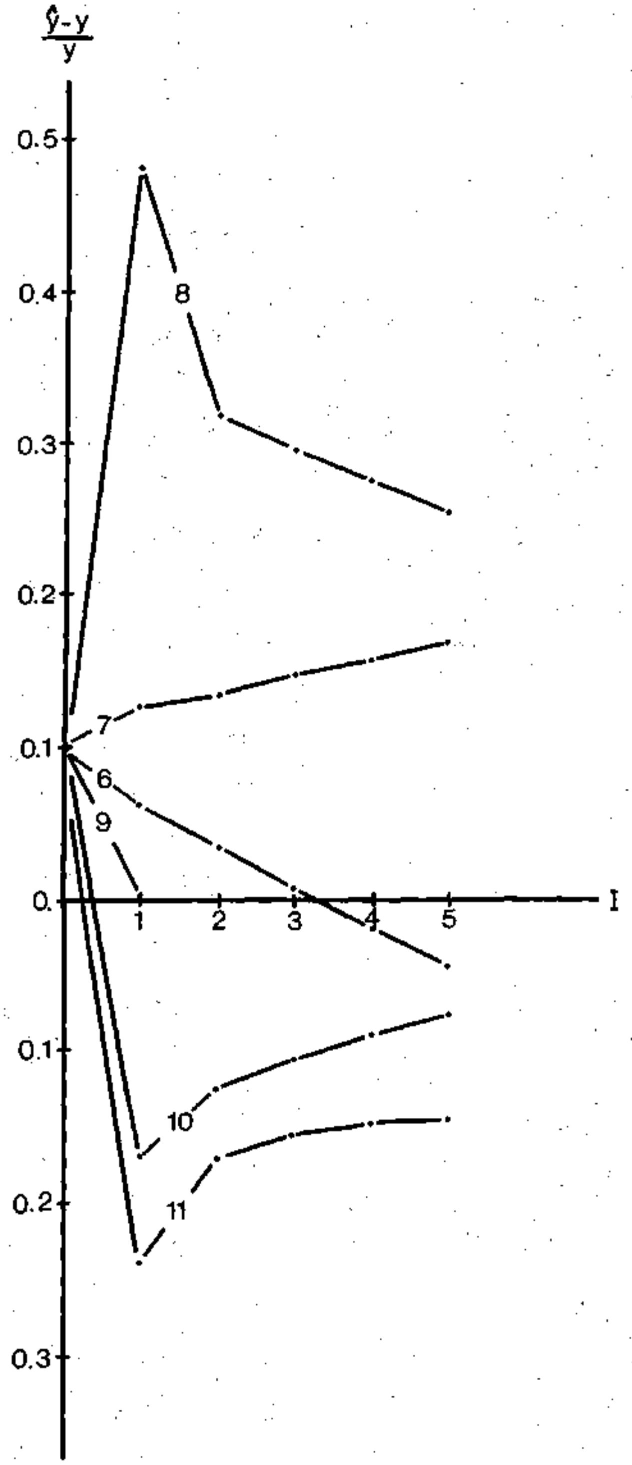


Figure 6b. Parameter Error Versus Iteration for Test Four, Parameters 6-11

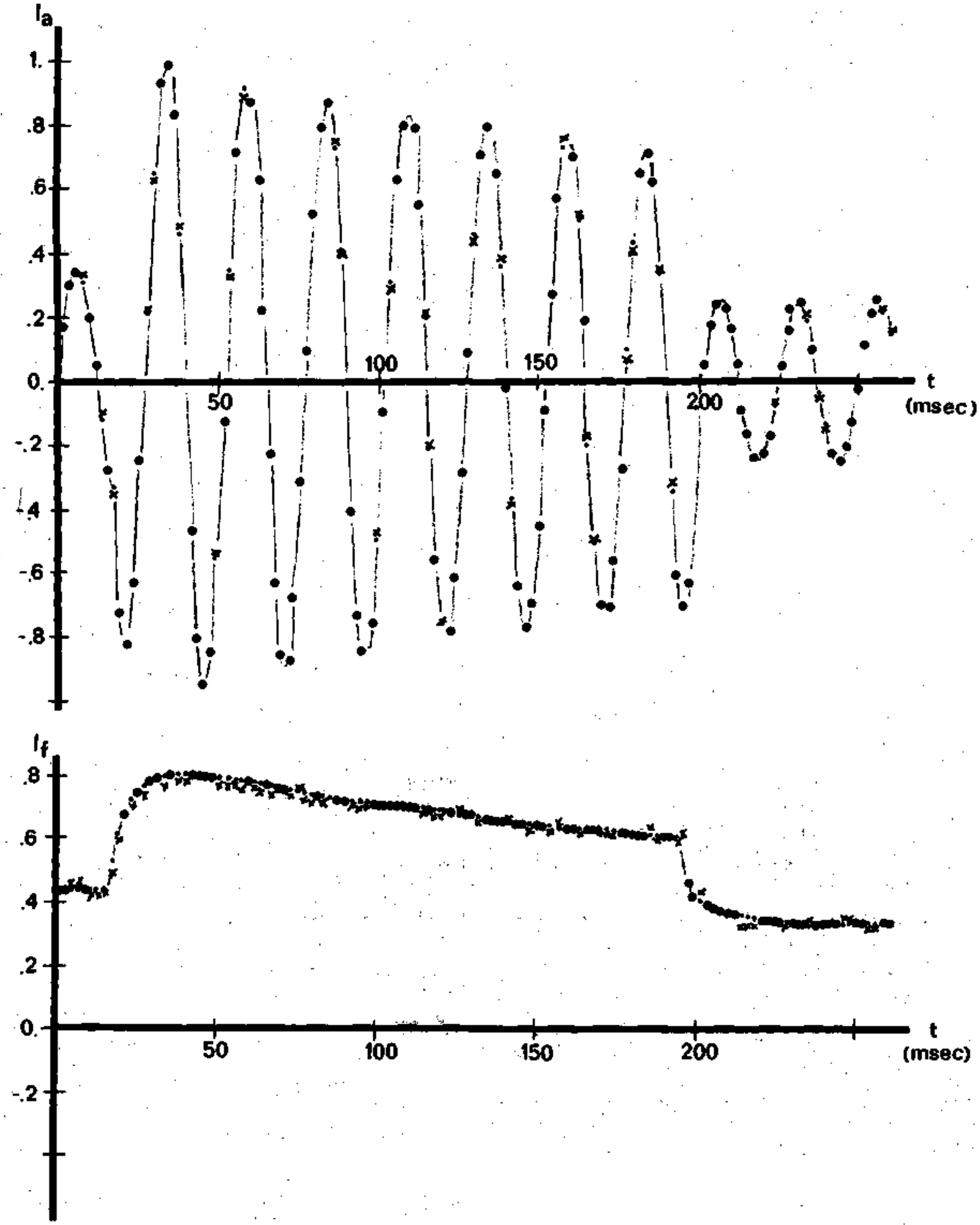


Figure 6c. Armature and Field Current for Test Four

* = outputs based on parameter estimate
✖ = input data
● = points where input matches output within graphical error

CHAPTER IV

IMPLEMENTATION OF THE EXPERIMENT

Introduction

This chapter describes the experiment to measure the parameter of the synchronous generator in the laboratory. Terminal current measurements are recorded, stored in digitized form, and fed off-line into the parameter estimation algorithm. The resulting parameter estimates are used to predict new results, which are compared to actual measurements to validate the model.

The three main thrusts of this part of the research are reported in the remainder of this chapter. First the data collection, processing and storage methods are described in detail. Next, the results of the experiment, the parameter estimates, are presented. Finally, the adequacy of the results is assessed by using the parameter estimates in a computer simulation to predict the generator response to a relatively large change in load. This prediction is compared to actual measurements under the same conditions.

Data Processing Methods

Analog signals proportional to the four terminal currents are produced by current shunts in the four generator terminal circuits. These signals are recorded on separate channels of an FM instrumentation tape recorder. A typical channel is shown schematically in Figure 7.

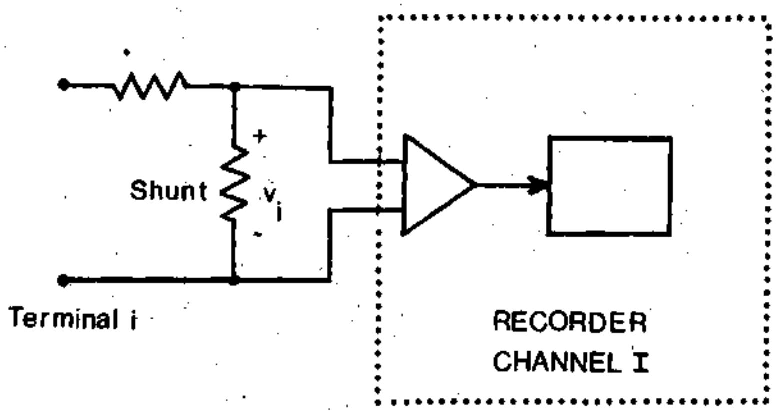


Figure 7. Schematic Diagram of Instrumentation for Channel i.

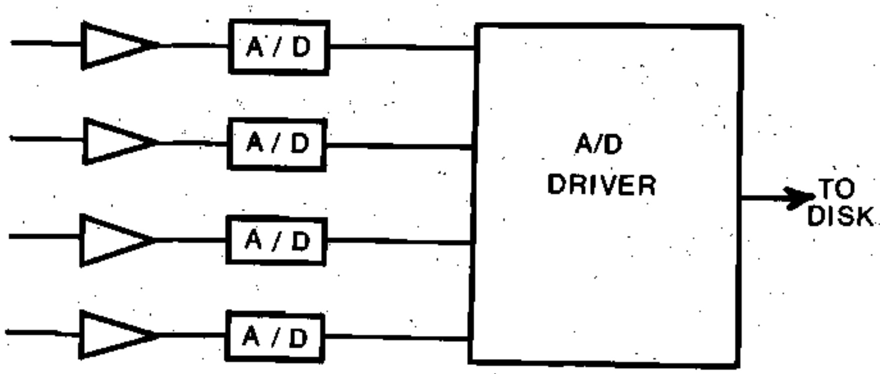


Figure 8. Four Channels of Data Digitized and Written to Disk Serially.

Since the parameter estimator is implemented on a digital computer, the four channels of analog data are digitized off-line and stored in serial form on a magnetic disk cartridge. Figure 8 is a schematic diagram of the analog-to-digital (A/D) conversion and the multiplexing of the four digital data channels into one serial record on the disk. The A/D converters provide sampled-data output in 12 bit two's complement binary numbers. The A/D driver software writes these numbers to the disk serially. The stream of data to disk contains sample one of channel one, then sample one of channel two and so forth. Sample two of channel one follows sample one from the last channel. This process of multiplexing data to the disk is illustrated for two channels of triangular waveform data in Figure 9.

Thus the data stored on the disk forms a time series

$$z_1(t_k),\ z_2(t_k + \frac{\Delta T}{4}),\ z_3(t_k + \frac{2\Delta T}{4}),\ z_4(t_k + \frac{3\Delta T}{4}),$$

$$z_1(t_{k+1}),\ z_2(t_{k+1} + \frac{\Delta T}{4}),\ .\ .\ .\ ,z_4(t_f)\ .$$

Here ΔT is the overall time step and $t_{k+1} = t_k + \Delta T$. Data in this serial multiplexed format is used directly in a weighted least-squares algorithm by updating the state vector and the sensitivity matrices at all the time points

$$t_k,\ t_k + \frac{\Delta T}{4},\ t_k + \frac{2\Delta T}{4},\ t_k + \frac{3\Delta T}{4},\ t_{k+1},\ .\ .\ .\ ,t_f.$$

The error between observed and computed outputs is defined as

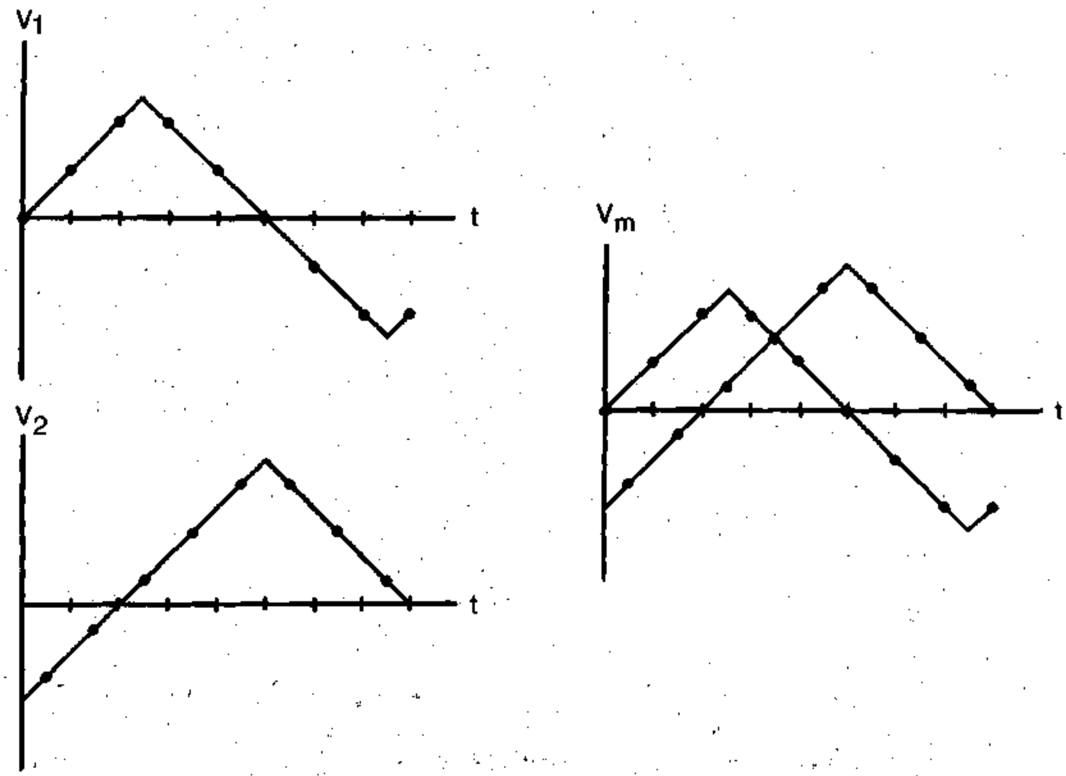


Figure 9. Reduction of Two Channels of Digitized Data to One Serial Record. (a) Two Channels of Data Sampled Alternately, (b) Multiplexed Serial Data.

$$\tilde{z}(k) \triangleq \begin{bmatrix} z_1(t_k) \\ z_2(t_k + \frac{\Delta T}{4}) \\ z_3(t_k + \frac{2\Delta T}{4}) \\ z_4(t_k + \frac{3\Delta T}{4}) \end{bmatrix} - \begin{bmatrix} h_1[x(t_k), \hat{y}, t_k] \\ h_2[x(t_k + \frac{\Delta T}{4}), \hat{y}, t_k + \frac{\Delta T}{4}] \\ h_3[x(t_k + \frac{2\Delta T}{4}), \hat{y}, t_k + \frac{2\Delta T}{4}] \\ h_4[x(t_k + \frac{3\Delta T}{4}), \hat{y}, t_k + \frac{3\Delta T}{4}] \end{bmatrix} \tag{53}$$

This new definition is used to implement the estimator directly from the data on disk.

Instrumentation

The instrumentation used in this research includes an ac ammeter, an ac voltmeter, and a digital multimeter for dc measurements. These meters are listed with model numbers and manufacturers in Table 2. In addition to these instruments for steady-state measurements, a Honeywell model 5600 instrumentation FM tape recorder was used to record transient data. Analog-to-digital conversion and data processing, described previously, were implemented on a Data General NOVA small computer.

Result of the Experiment

After operating the generator at normal load for fifteen minutes to minimize variations due to temperature changes, the load resistance was measured as 0.98 per unit. The additional load resistor bank was switched in parallel to this load, and the resistance of the combination measured as 0.23 per unit. Then the load was returned to normal, and

Table 2. Parameter Error Weights

ITERATION	PARAMETERS	WEIGHT
1-13	Y_1-Y_7	$1x10^7$
1-3	Y_8-Y_{11}	$1x10^6$
4-13	Y_8, Y_9	$1x10^5$
4-13	Y_{10}, Y_{11}	$1x10^4$

the rotor speed measured as 252 radians per second. The first test was performed under these conditions. The resulting data, recorded and processed as previously described, was sampled at a rate of 500 Hertz per channel or an overall rate of 2KHz for all four channels.

To determine a priori information about the measurement noise, the recorder was run with the inputs shorted to ground. After digitizing, this data indicated approximately equal noise variances in all four channels. As a result the weights for the measurements were selected as unity. Due to lack of good a priori information on the initial parameter error variances, these weights were adjusted empirically to obtain good results from the estimator. By weighting the parameters heavily at first, estimates were prevented from large excursions on the first few iterations, enhancing the stability of the estimator. The weights chosen are summarized in Table 3, while the parameter estimates are plotted versus iteration number in Figure 10.

Figure 11 shows a comparison of the phase A current and the field current predicted from the parameter estimates to the corresponding data from the experiment. First, in Figure 11(a), the results of the initial parameter estimates are compared to the experimental data. Then Figure 11(b) shows the results of the final parameter estimates compared to the data. The final results follow the data quite well. This indicates that the model fits the data well at these particular operating conditions.

To test these results under another condition and to assess the validity of the model, a second test was run with load resistance switched from 0.99 per unit to 0.175 per unit. This test was first

Table 3. List of Instruments Used

Description	Number	Manufacturer
AC ammeter	AA-10	General Electric
AC voltmeter	AV-12	General Electric
Digital multimeter	3476B	Hewlett-Packard

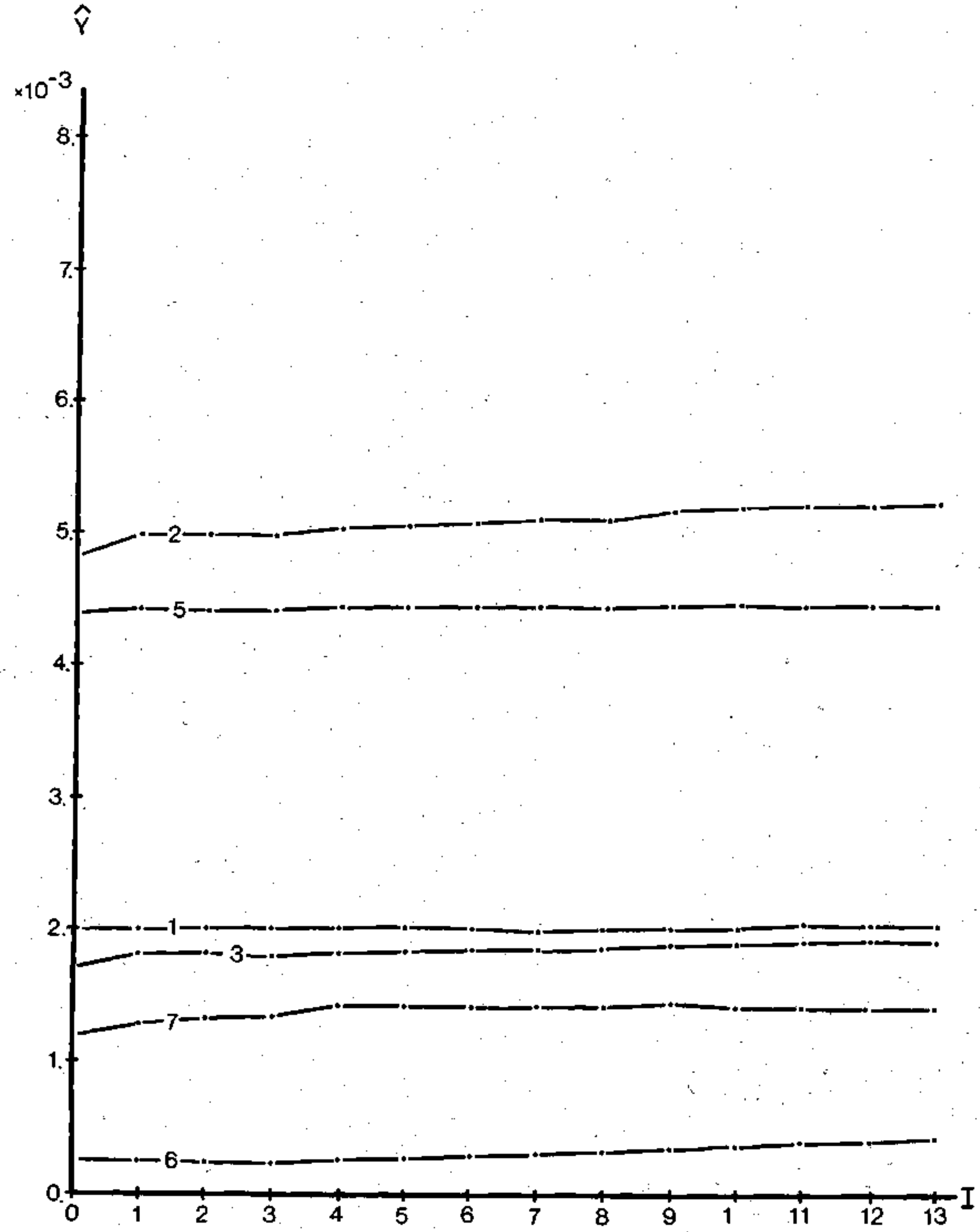


Figure 10a. Parameter Estimates Versus Iteration Number from Experimental Data (Test One), Parameters 1, 2, 3, 5, 6, and 7

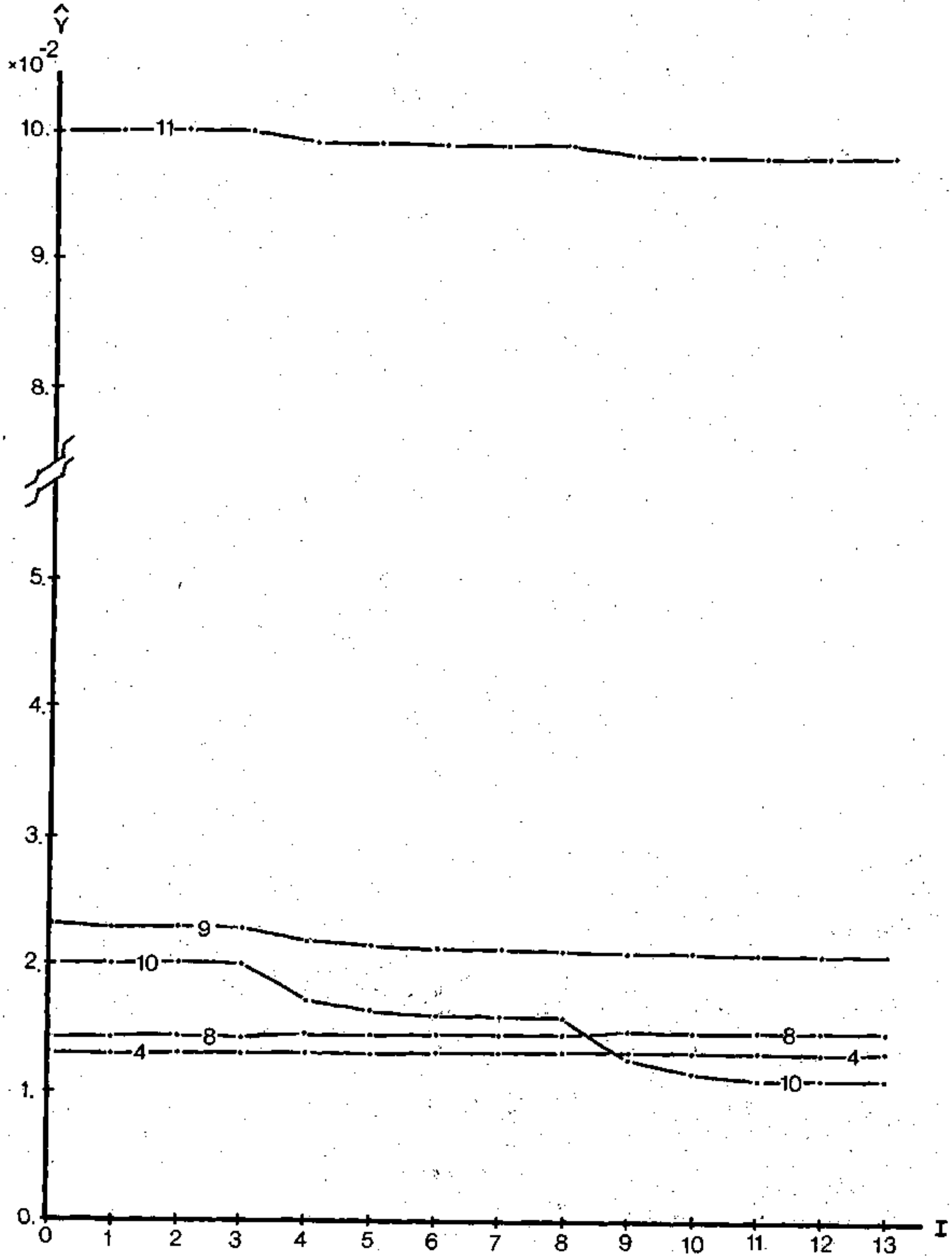


Figure 10b. Parameter Estimates Versus Iteration Number from Experimental Data (Test One), Parameters 4, 8, 9, 10, and 11

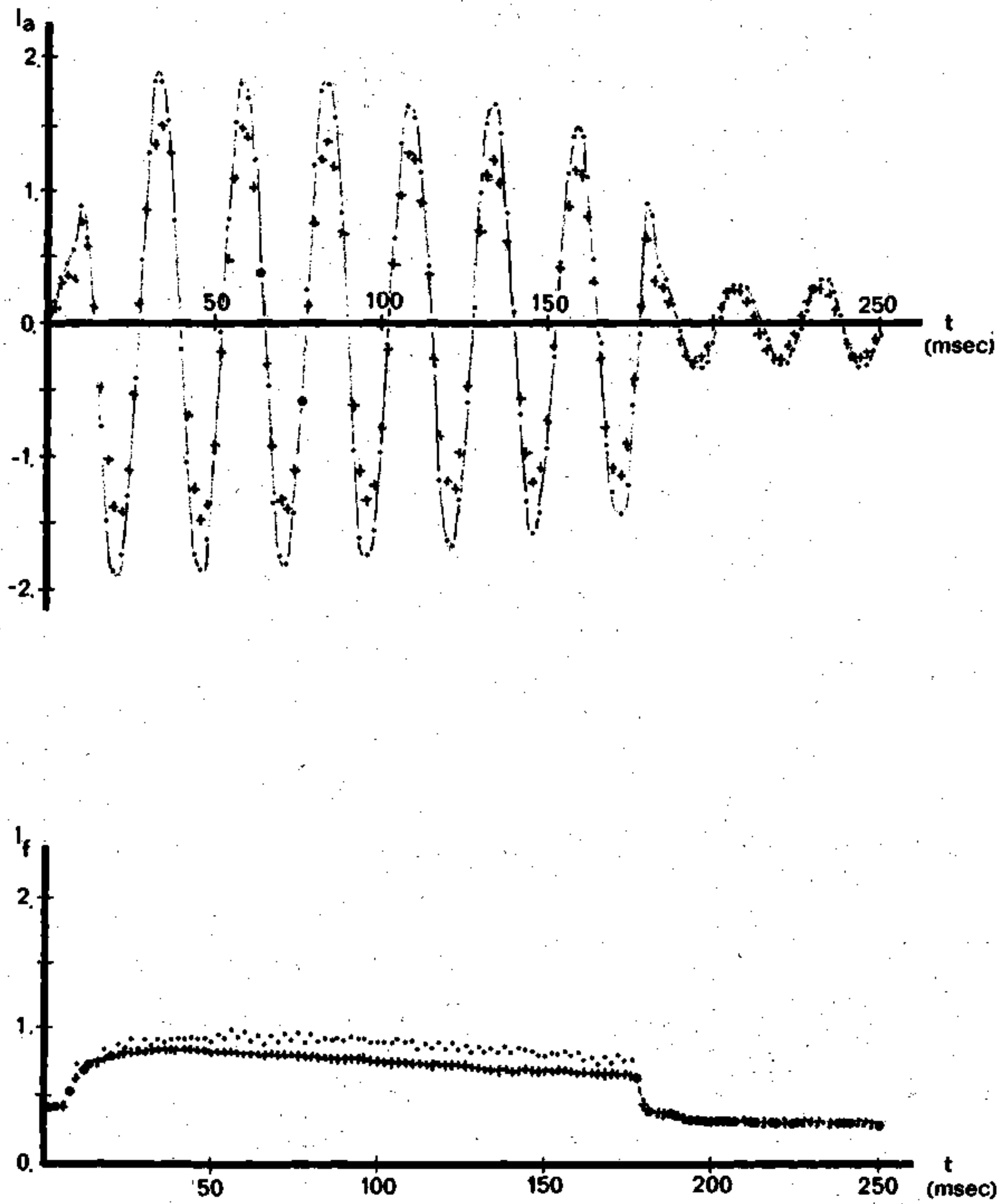


Figure 11a. Armature and Field Currents for Test One, Iteration Zero.

· = data from test one
+ = predicted from initial parameter estimates

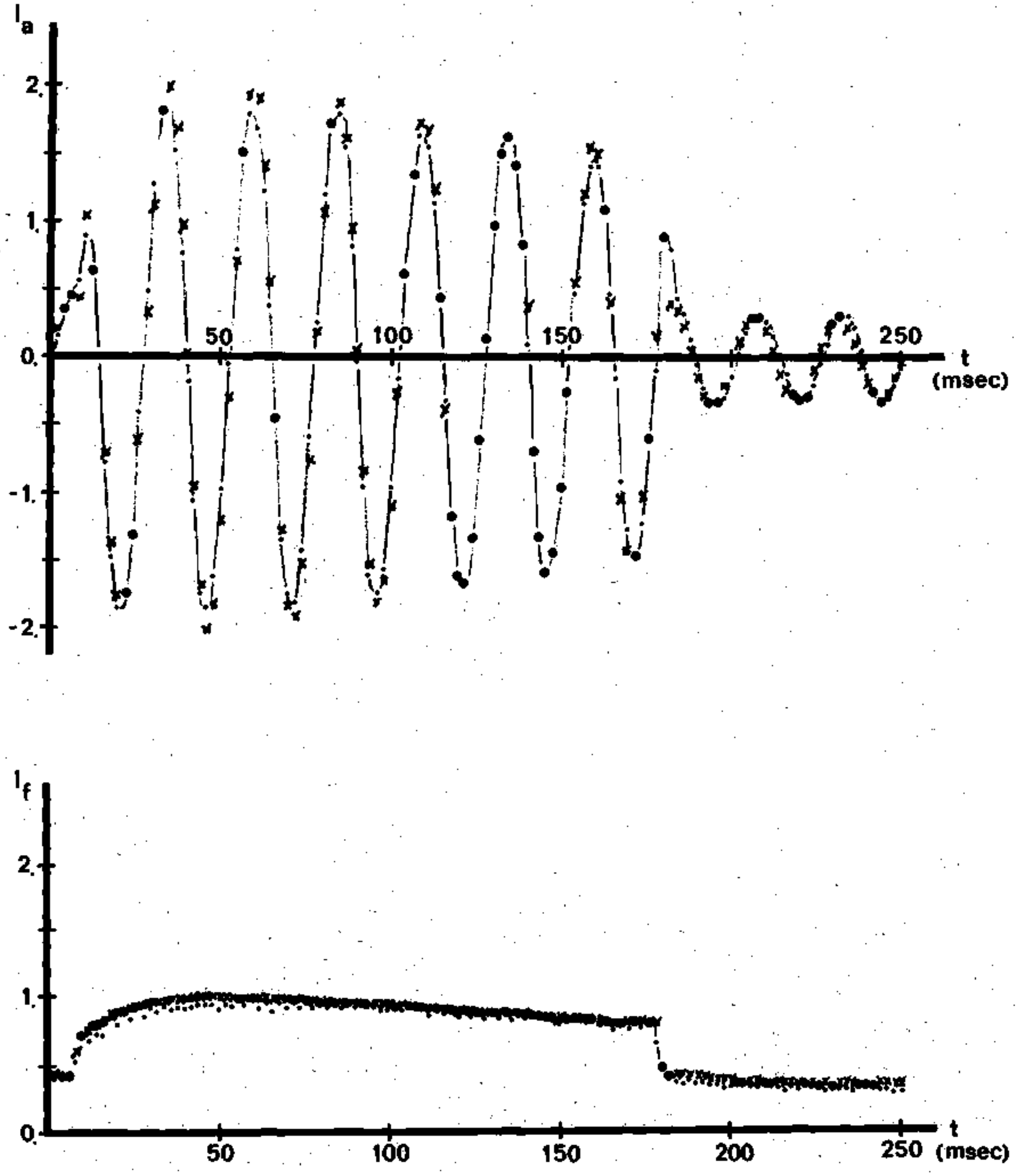


Figure 11b. Armature and Field Currents for Test One, Iteration 13.

• = data from test one
x = predicted from final parameter estimates
● = data and estimates coincide within graphical errors

simulated on the digital computer using the parameter estimates from the first test, then implemented in the laboratory. Figure 12 shows the comparison of the simulated results to the experimental results. The reasonable agreement of these results indicates the success of the model of the synchronous generator at similar operating conditions. This does not, however, guarantee validity of this model over much larger changes in operating conditions. In fact, a synchronous generator exhibits nonlinearities; therefore, this linearized representation is probably somewhat inaccurate for very large perturbations. The model is valid, though, about the conditions of the tests.

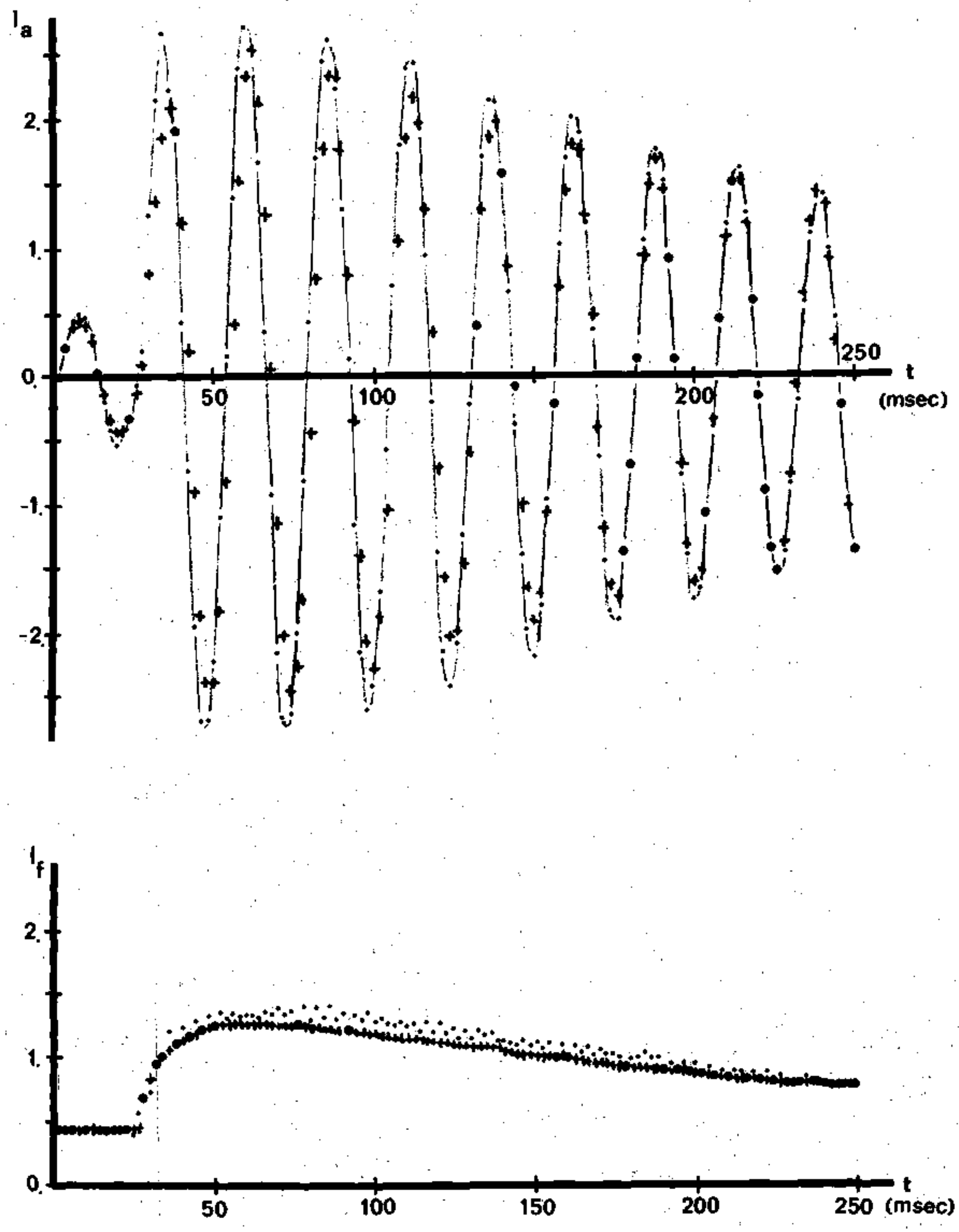


Figure 12. Armature and Field Currents.

· = data from test two
+ = prediction from parameter estimates of test one
• = data and estimates coincide within graphical error

CHAPTER V

CONCLUSIONS

Summary of Results

This work is a new approach to an old problem: system identification theory applied to the experimental determination of synchronous generator parameters. The power of this statistical identification technique allows the parameters of Park's equations, cast in a state-variable formulation, to be determined directly from terminal current measurements.

Considering the identification problem as a multipoint boundary-value problem and applying the method of quasilinearization leads to a weighted least-squares parameter estimator. Including additive noise corrupting the measurement process, a statistical analysis proves the maximum a posteriori probability estimator is achieved by selecting the weighting matrices as the inverse noise and parameter error covariance matrices. This estimator is the optimal estimator in the sense of minimizing the Bayesian risk. If the error covariances are incorrect, the estimator is no longer optimal but is still a good implementation of a least-squares estimator.

Before the experiment was implemented, several experimental conditions, such as the magnitude and duration of the transient and the data sampling rate and duration, needed to be selected. Also a set of constraints, mainly economic in nature, were recognized. Within these constraints the experiment was designed with the aid of a digital computer simulation. That is, a numerical solution of the mathematical model of a typical synchronous generator under a switched resistive load, was used to test various experimental conditions. The results show that a sufficient amount of data is produced by a load resistance switched from full load value to 0.25 per unit then switched back to full load value. The duration of the transient was consistent with manual operation of a three-pole switch. The resulting experiment provided sufficient data at a sampling rate which could be handled by the data recording and processing facilities available, while being implemented on available laboratory equipment.

Implementing this experiment in the laboratory, recording the terminal current data, and digitizing this data for input to the off-line parameter estimator led to estimates of the parameters of the mathematical model of the generator. These parameter estimates provide a linearized model of the generator which is valid over a range of operating conditions near the experimental conditions. The results of predicting the generator response to a larger step change in load resistance compare favorably to the actual measured response under those conditions. This favorable comparison validates the results, showing that the model can indeed predict the generator behavior under similar load conditions.

Significance of Results

In fact, the synchronous generator is a nonlinear device, exhibiting saturation and other deviations from the assumed linear model. By assuming this linear model structure and using an iterative estimation algorithm to calculate parameters, the results are effectively linearized about the test conditions. That is, the estimator fits the best linear model to the data. This, of course, presents a limitation to modeling drastically different operating conditions. For relatively small perturbations in conditions, however, a simple valid solution has been achieved. It should be emphasized that the approach used here is an improvement over methods which assume linearity of the model over large changes in operating condition. For example, estimates of X_d, the synchronous reactance, can be obtained from combining results of open-circuit, unsaturated, armature voltage and of short-circuit armature current. However, such tests assume that results of two tests at drastically different points can be combined to model the generator under still different conditions, such as at unity power factor and full load. This implies that the generator model is linear to large perturbations. Thus, the present work should be viewed as a step toward proper modeling of a nonlinear device by a linear model valid over a small operating region.

The assumption of a linear model was made primarily for simplicity. The estimation of parameters of a dynamical system, such as the generator, can be viewed as an inherently nonlinear problem, even when the model itself is linear. Augment the state vector x with the parameter vector y,

$$\tilde{x} = \begin{bmatrix} x \\ \hline y \end{bmatrix} . \tag{54}$$

Now, in the linear model used, the parameters are multiplied by the states in the right side of equation (24). As a result, this can be rewritten as a nonlinear dynamical system in equation (55).

$$\frac{d\tilde{x}}{dt} = f(\tilde{x}) \tag{55}$$

The problem can now be formulated as a nonlinear state estimation problem. As a result of this view of the problem, any nonlinearity that can be formulated can be included by this approach.

Therefore, a general method for determination of synchronous generator parameters from the results of test data has been presented. The particular model used here is Park's linear circuit model of an ideal generator. A solution, using available equipment, shows favorable results.

Recommendations for Future Research

Inclusion of nonlinearities, especially saturation, in the generator model deserves additional research. The preceding section shows that nonlinear differential equations are easily handled by this algorithm. Consequently, future research should concentrate on finding a suitable parameterization of the nonlinearities. Once this is done, the extension of the present algorithm to include estimating the parameters that describe the nonlinearities is trivial.

APPENDIX A

LOWER BOUND ON ERROR COVARIANCE

The inverse of Fisher's information matrix provides a lower bound on the error covariance of the maximum likelihood estimate [16,13]. A similar approach results in a lower bound on the error covariance of the maximum a posteriori estimate [15]. These bounds are generalizations of the Cramer-Rao bound. This appendix outlines derivation of this bound for the estimator used in this research.

If the estimator were to converge exactly to a parameter estimate $\tilde{y}$, then

$$\sum_{k=1}^{k_f}\left\{\frac{dh(t_k)}{dy}^T\bigg|_{\tilde{y}} V_w^{-1}[z(t_k) - h(x,\tilde{y},t_k)]\right\} - V_y^{-1}(\tilde{y}-m_y) = 0 \quad . \tag{A.1}$$

In words, the model output $h(x,\tilde{y},t_k)$ would exactly equal the observed output $z(t_k)$. This condition is the theoretical best estimate of the parameter vector, but is never achieved in the presence of measurement noise. It is reasonable to use this theoretical performance limit to obtain a lower bound on the error covariance. Linearizing $h(x,\tilde{y},t_k)$ about the true parameter vector, y, gives equation (A.2)

$$\sum_{k=1}^{k_f}\left\{\frac{dh(t_k)}{dy}^T\bigg|_{\tilde{y}} V_w^{-1}[z(t_k) - h(x,y,t_k) - \frac{dh}{dy}\bigg|_{\tilde{y}}(\tilde{y}-y)]\right\}$$

$$-V_y^{-1}(\tilde{y}-y) - V_y^{-1}(y-m) = 0 \quad . \tag{A.2}$$

Solving for $(\tilde{y}-y)$ gives Equation (A.3).

$$(\tilde{y}-y) = R^{-1}\{\sum_{k=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1}(w_k)] - V_y^{-1}(y-m_y)\} \tag{A.3}$$

where

$$R = \{\sum_{k=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1} \frac{dh(t_k)}{dy}\Big|_{\tilde{y}}] + V_y^{-1}\} \tag{A.4}$$

and

$$w_k = z(t_k) - h(x,y,t_k) \quad .$$

Squaring and taking expected value yields the desired error covariance, Equation (A.5).

$$\begin{aligned} \text{cov}(\tilde{y}-y) = R^{-1}\{ & \sum_{k=1}^{k_f}\sum_{j=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1}E(w_k w_j^T)V_w^{-1} \frac{dh(t_j)}{dy}\Big|_{\tilde{y}} \\ & -2\sum_{k=1}^{k_f} \frac{dh(t_k)}{dy}^T\Big|_{\tilde{y}} V_w^{-1}E[w_k(y-m_y)^T]V_y^{-1} + V_y^{-1}\}R^{-1} \quad , \end{aligned} \tag{A.5}$$

Now the noise samples are assumed independent

$$E(w_k w_j^T) = \begin{cases} V_w & \text{if } k=j \\ 0 & \text{if } k\neq j \end{cases} \quad , \tag{A.6}$$

and $E[w_k(y-m)^T]$ is assumed to be zero, i.e. the noise is assumed uncorrelated with the random parameter vector. As a result, the desired bound is simply

$$\mathrm{cov}(\tilde{y}-y) = R^{-1}RR^{-1} = R^{-1} \quad . \tag{A.7}$$

APPENDIX B

NOMINAL PARAMETERS FOR SIMULATION

The nominal parameters used in the simulated experiment of Chapter II were approximations derived from nameplate data, steady-state measurements, and a list of typical machine parameters [3].

From the generator nameplate the rated values of armature voltage and current were

$$v_B = \frac{230}{\sqrt{3}} = 132.79 \text{ volts, line to neutral} \tag{B.1}$$

and

$$i_B = \frac{1000}{132.79} = 7.531 \text{ amps} \quad . \tag{B.2}$$

The angular speed of the equivalent two-pole machine is

$$\omega_o = 1200 \times \frac{2\pi}{60} \times 2 = 251.33 \text{ radians/second} \quad . \tag{B.3}$$

The results of steady-state open-circuit and short-circuit tests are plotted in Figure (B.1). The rated value of open-circuit armature voltage corresponds to

$$i_{FB} = .31 \text{ amps,} \tag{B.4}$$

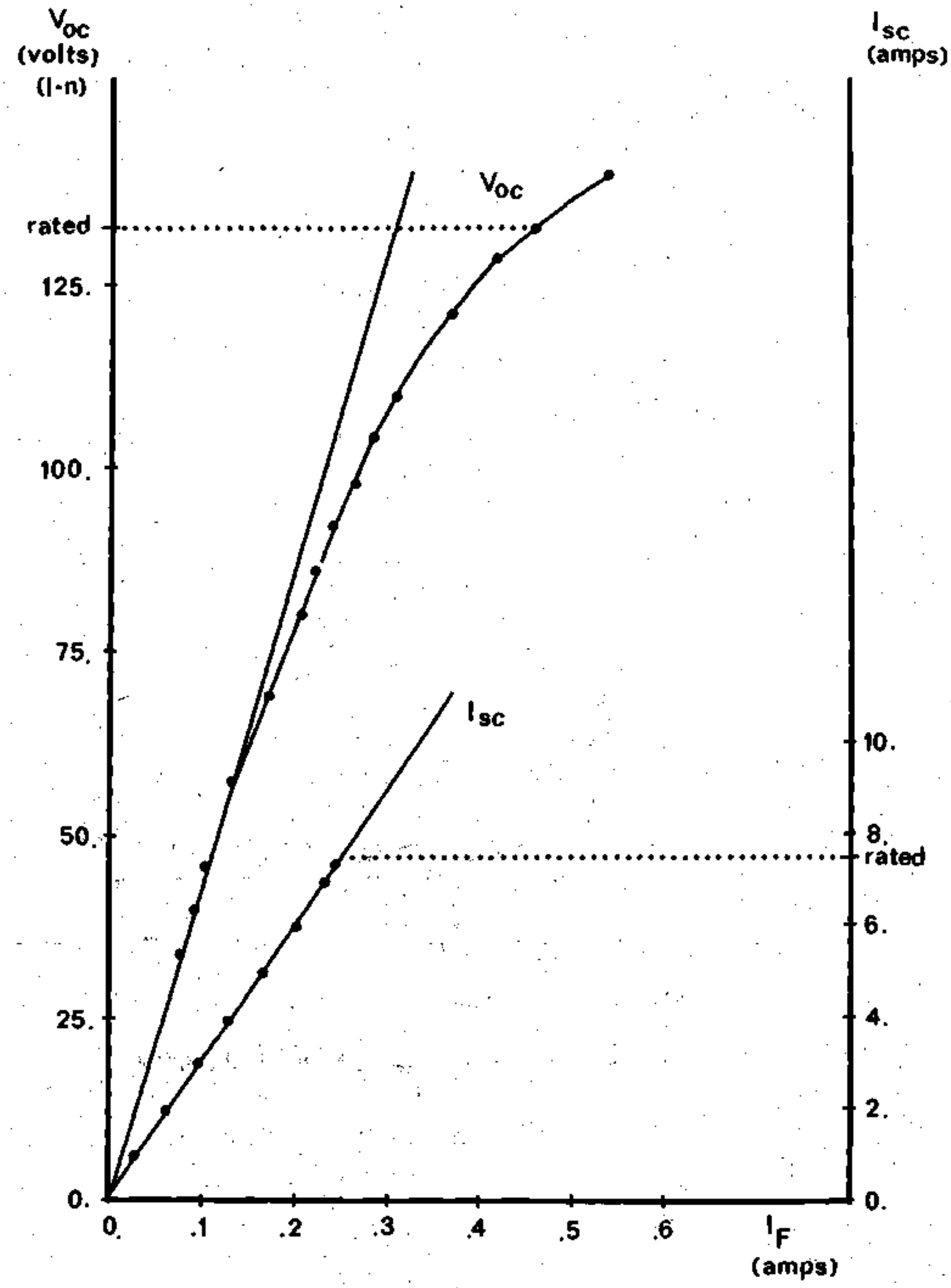


Figure B1. Open-Circuit Voltage and Short-Circuit Current Versus Field Current.

neglecting saturation. Thus,

$$v_{FB} = \frac{v_B i_B}{i_{FB}} = 3226 \text{ volts} \tag{B.5}$$

and

$$L_{df} \cong 4.87\text{x}10^{-3} \text{ per unit} \quad . \tag{B.6}$$

Rated short-circuit current corresponds to

$$i_{FS} = .253 \text{ amps} \quad ; \tag{B.7}$$

therefore,

$$L_d \cong \frac{i_{FS}}{i_{FB}} \frac{1}{w_o} = 3.25\text{x}10^{-3} \text{ per unit} \quad . \tag{B.8}$$

A typical value of transient reactance is

$$X_d' = .322\ X_d \quad . \tag{B.9}$$

Therefore, from the definition of transient reactance,

$$X_d - X_d' = \frac{X_{df}^2}{X_f} \quad , \tag{B.10}$$

and

$$L_f \simeq \frac{L_{df}^2}{.678L_d} = 10.78\text{x}10^{-3} \text{ per unit} \quad . \tag{B.11}$$

The remaining d-axis parameters are roughly approximated by assuming equal coupling from stator to all rotor circuits.

$$L_{kd} = k^2L_d = .678\text{x}3.25\text{x}10^{-3} = 2.20\text{x}10^{-3} \text{ p.u.} \tag{B.12}$$

$$L_{fkd} = \sqrt{k^2L_fL_{kd}} = \sqrt{k^2L_{df}^2} = kL_{df} = 4.01\text{x}10^{-3} \text{ p.u.} \tag{B.13}$$

Typical q-axis parameters give

$$L_q \simeq .652L_d = 2.12\text{x}10^{-3} \text{ per unit} \tag{B.14}$$

and

$$L_{kq} = L_q - L_q'' = .55\text{x}2.12\text{x}10^{-3} = 1.17\text{x}10^{-3} \text{ per unit} \tag{B.15}$$

With the rotor at standstill, direct current measurements give

$$R_D = .25\Omega \qquad \text{and} \qquad R_F = 240\Omega \quad , \tag{B.16}$$

or

$$R_d = 1.418\text{x}10^{-2} \text{ p.u.} \quad \text{and} \quad R_f = 2.306\text{x}10^{-2} \text{ p.u.} \tag{B.17}$$

The damper resistances are not measurable; however, typical time constants are

$$T_{do}' = \frac{L_f}{R_f} = .467 \text{ sec.} \tag{B.18}$$

and

$$T_{do}'' \cong .02\ T_{do}' = .00935 \text{ sec.} \tag{B.19}$$

The result is

$$R_{kd} \cong \frac{L_{kd} - \dfrac{L_{fkd}^2}{L_f}}{T_{do}''} = .0759 \text{ p.u.} \tag{B.20}$$

For lack of better information, we assume

$$R_{kq} \cong R_{kd} \quad . \tag{B.21}$$

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